OSDAV Public School, Kaithal

First Unit Test (May, 2024)

Class : XII

Subject : Mathematics (Core) Set-A

Time: 1 hr 30 min. **M.M. : 30**

General Instructions:-

All questions are compulsory.

(a) There are 20 questions in this question paper.

(b) SECTION A consists of 12 Multiple Choice questions.

(c) SECTION B consists of 6 questions carrying 2 marks each.

(d) SECTION C consists of 2 questions carrying 3 marks each.

Section A

1. Let A = {1, 2, 3}. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is:

a. 1 b. 2 c. 3 d. 4

2. A function $f : R \to R$ defined by $f(x) = 2 + x^2$ is:

a. Not one-one b. one-one c. not onto d. neither one-one onto

3. Let R be the relation "is congruent to" on the set of all triangles in a plane is:

a. reflexive only b. symmetric only

c. symmetric and reflexive only d. equivalence relation

4. If a matrix A is both symmetric and skew symmetric, then A is necessarily a:

a. diagonal matrix b. zero square matrix

c. square matrix d. identity matrix

5. If $A^2 = A$, then $(A + I)^4$ is equal to:

a. I + A b. I + 4A c. I + 15A d. None of these

6. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ then:

a. $A^{-1} = B$ b. $A^{-1} = 6B$ c. $B^{-1} = B$ d. $B^{-1} = \frac{1}{6} A$

7. If $A = \begin{bmatrix} 4 & 1 & 0 \\ 1 & -2 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & x \end{bmatrix}$ $C = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $D = \begin{bmatrix} 15 + x \\ 1 \end{bmatrix}$ such that $(2A - 3B)\,C = D$, then x =

a. 3 b. -4 c. -6 d. 6

8. If A is 3 x 3 matrix such that |A| = 8, then |3A| equals to:

a. 8 b. 24 c. 72 d. 216

9. Find cofactors of a_{21} and a_{31} of the matrix: $A = [a_{ij}] = \begin{bmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{bmatrix}$

a. -16, 8 b. -16, -8 c. 16, 8 d. 16, -8

10. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then value of x is :

a. 3 b. ± 3 c. ± 6 d. 6

11. Value of k, for which $A = \begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix is:

a. 4 b. -4 c. $\pm$ 4 d. 0

12. If A is a square matrix of order 3 and |A| = -5, then |adj A| is:

a. 125 b. -25 c. 25 d. ± 25

Section B

13. Show that the signum function f : R → R, given by: $f(x) = \begin{cases} 1, & if\ x > 0 \\ 0, & if\ x = 0 \\ -1, & if\ x < 0 \end{cases}$

is neither one-one nor onto.

14. If the function f : Q → Q is defined by f(x) = 3x – 4 ∀ x ∈ Q, then show that *f* is one-one and onto, where Q is the set of rational numbers.

15. Let $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$, then find a matrix D such that CD – AB = 0.

16. Express the following matrix as the sum of a symmetric and skew symmetric matrix $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$

17. Show that the points (a, b + c), (b, c + a) and (c, a + b) are collinear, using determinants.

18. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$, then show that $(AB)^{-1} = B^{-1} A^{-1}$.

Section C

19. Prove that every square matrix can be expressed as the sum of a symmetric and a skew-symmetric matrix.

20. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}.

Using A^{-1}, solve the following system of equiations:

2x – 3y + 5z = 11, 3x +2y – 4z = -5, x+y -2z = -3

OSDAV Public School, Kaithal
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Class : XII
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Section A

1. Let A = {1, 2, 3}. Then number of equivalence relations containing (1, 2) is:
a. 1 b. 2 c. 3 d. 4

2. A function $f : R \to R$ defined by $f(x) = 2 + x^2$ is:
a. Not one-one b. one-one c. not onto d. neither one-one onto

3. Let R be the relation "is congruent to" on the set of all triangles in a plane is:
a. reflexive only b. symmetric only
c. symmetric and reflexive only d. equivalence relation

4. If A and B are symmetric matrices of the same order, then:
a. AB is a symmetric matrix b. A-B is a skew-symmetric matrix
c. AB + BA is a symmetric matrix d. AB –BA is a symmetric matrix

5. If $A^2 = A$, then $(A + I)^4$ is equal to:
a. I + A b. I + 4A c. I + 15A d. None of these

6. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ then:
a. $A^{-1} = B$ b. $A^{-1} = 6B$ c. $B^{-1} = B$ d. $B^{-1} = \frac{1}{6} A$

7. If $A = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$ and $A^2 - 4A + 10I = A$ then k is equal to:
a. 0 b. -4 c. 4 and not 1 d. 1 or 4

8. If A is 3 x 3 matrix such that $|A| = 8$, then $|3A|$ equals to:
a. 8 b. 24 c. 72 d. 216

9. Find cofactors of a_{21} and a_{31} of the matrix: $A = [a_{ij}] = \begin{bmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{bmatrix}$
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12. If A is a square matrix of order 3 and |A| = -5, then |adj A| is:

a. 125 b. -25 c. 25 d. $\pm$25

Section B

13. Show that the signum function f : R → R, given by: f(x) = $\begin{cases} 1, & if\ x > 0 \\ 0, & if\ x = 0 \\ -1, & if\ x < 0 \end{cases}$

is neither one-one nor onto.

14. If the function f : Q → Q is defined by f(x) = 3x – 4 ∀ x ∈ Q, then show that *f* is one-one and onto, where Q is the set of rational numbers.

15. Find the value of x from the following:

$$[\,x \quad -5 \quad -1\,]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

16. Express the following matrix as the sum of a symmetric and skew symmetric matrix $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$

17. Show that the points (a, b + c), (b, c + a) and (c, a + b) are collinear, using determinants.

18. If A = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$, then show that $(AB)^{-1} = B^{-1} A^{-1}$.

Section C

19. Prove that any square matrix A is invertible if and only if it is non singular.

20. If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}.

Using A^{-1}, solve the following system of equiations:

2x – 3y + 5z = 11, 3x +2y – 4z = -5, x+y -2z = -3

May Unit Test [Set - A] Class:- XII

Mathematics (Core)

Marking Scheme / Hints to solution

Note:- Any other relevant answer not given here in but given by the students, be suitably awarded.

Q No.	Value Points / Key points	Marks alloted to each key point	Total Points
	Section A		
1	(a) 1	1	1
2	(d) neither one one nor onto	1	1
3	(d) equivalance relation	1	1
4	(b) zero matrix	1	1
5	(c) $I + 15A$	1	1
6	(d) $B^{-1} = \frac{1}{6} A$	1	1
7	(c) $x = -6$	1	1
8	(d) 216	1	1
9	(a) $-16, 8$	1	1

10	(c) $x = \pm 6$	1	1
11	(c) $k = \pm 4$	1	1
12	(c) 25	1	1

Section B

13

$$f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

<u>One - one</u>

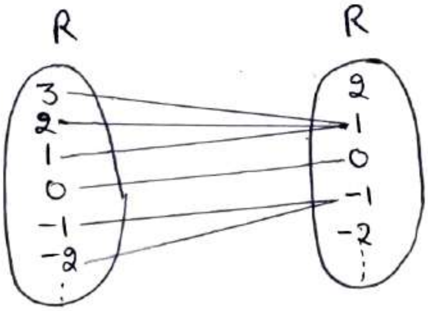
$2, 3 \in R$

$f(2) = 1$ as $2 > 0$

$f(3) = 1$ as $3 > 0$ — 1

As different elements have same image so $f(x)$ is not one - one. — 2

<u>Onto</u>



1

As -2 does not have any pre-image so, Range $\neq$ codomain

$\therefore f(x)$ is not onto

Hence Proved

14 $f(x) = 3x-4$

Let $x, y \in Q$ (Domain)

$$f(x) = f(y)$$

$$3x - \cancel{4} = 3y - \cancel{4}$$

$$\cancel{3}x = \cancel{3}y$$

$$x = y$$

$\therefore$ $f(x)$ is one-one function 1

Let $f(x) = y$ such that $y \in Q$ (co-domain) 2

$$f(x) = y = 3x-4$$

$$x = \frac{y+4}{3} \in Q \text{ (Domain)}$$

It is true for every $y = 3x-4$

because $f(x) = f\left(\frac{y+4}{3}\right) = \cancel{3}\left(\frac{y+4}{\cancel{3}}\right) - 4$

$$f(x) = y$$ 1

Therefore, it is an onto function

Hence Proved

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15 Let Matrix D is of order 2×2

such that $D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$$ 1/2

$$\begin{bmatrix} 2a+5c & 2b+5d \\ 3a+8c & 3b+8d \end{bmatrix} = \begin{bmatrix} 10+7 & 4+4 \\ 15+28 & 6+16 \end{bmatrix}$$

$$\begin{bmatrix} 2a+5c & 2b+5d \\ 3a+8c & 3b+8d \end{bmatrix} = \begin{bmatrix} 17 & 8 \\ 43 & 22 \end{bmatrix}$$ 1/2

$3 \times \quad 2a + 5c = 17$

$2 \times \quad 3a + 8c = 43$

$$\begin{array}{l} 6a + 15c = 51 \\ \underline{6a + 16c = 86} \\ - \quad - \quad - \\ \hline \cancel{-}c = \cancel{-}35 \\ \hline \end{array}$$

$$c = 35$$

$$2a + 5(35) = 17$$

$$2a = 17 - 175$$

$$\cancel{2}a = -\cancel{158}$$

$$a = -79$$ 1/2 2

$$3 \times \quad 2b + 5d = 8$$

$$2 \times \quad 3b + 8d = 22$$

$$\begin{array}{l} 6b + 15d = 24 \\ \underline{6b + 16d = 44} \\ - \quad - \quad - \\ \hline \cancel{-}d = \cancel{-}20 \\ \hline \end{array}$$

$$d = 20$$ 1/2

$$2b + 5(20) = 8$$

$$2b = 8 - 100$$

$$\cancel{2}b = -9\cancel{2}$$

$$b = -46$$

$$D = \begin{bmatrix} -79 & -46 \\ 35 & 20 \end{bmatrix}_{2\times 2}$$

16 $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$

$$A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$$

$$A + A' = \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$$

$$\frac{1}{2}(A + A') = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} \quad \text{--- } (1)$$ 1/2

This is symmetric matrix 2

$$A - A' = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

$$\frac{1}{2}(A - A') = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \quad \text{--- } (2)$$ 1/2

This is skew symmetric matrix

(1) + (2)

$$\begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = A$$ 1/2

Hence, $A = \underbrace{\frac{1}{2}(A + A')}_{\text{symmetric}} + \underbrace{\frac{1}{2}(A - A')}_{\text{skew-symmetric}}$ 1/2

---

17 $(a, b+c), (b, c+a), (c, a+b)$

$$= \begin{bmatrix} a & b+c & 1 \\ b & a+c & 1 \\ c & a+b & 1 \end{bmatrix}$$

$$\text{Area of } \Delta = \frac{1}{2} \begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$ 1/2 2

$$\frac{1}{2}\left(a(c+a-a-b) - (b+c)(b-c) + 1(ab + b^2 - c^2 - ac)\right)$$

	$= \frac{1}{2}\left(ac - ab - [b^2 - c^2] + ab + b^2 - c^2 - ac\right)$	1/2	
	$= \frac{1}{2}\left(\cancel{ac} - \cancel{ab} - \cancel{b^2} + \cancel{c^2} + \cancel{ab} + \cancel{b^2} - \cancel{c^2} - \cancel{ac}\right)$	1/2	2
	$= \frac{1}{2}(0) = 0$		
	Since, Area of $\Delta = 0$ These points are collinear	1/2	
18	$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$		
	$AB = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$		
	$Adj(AB) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}' = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$		
	$\frac{1}{\lvert AB \rvert} = \frac{1}{ad-bc}$		
	$AB^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$ — LHS	1/2	2
	$B^{-1} = \frac{1}{\lvert B \rvert} Adj\, B$		
	$Adj\, B = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$		
	$\lvert B \rvert = ad - bc$		
	$B^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$	1/2	
	$A^{-1} = \frac{Adj\, A}{\lvert A \rvert}$, $Adj\, A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$		
	$\lvert A \rvert = 1$		
	$A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$		
	$B^{-1}A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	1/2	

	$B^{-1}A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$ — RHS LHS = RHS Hence verified	1/2	
19	Let A be a square matrix Multiply and divide by 2 $A = \frac{1}{2}(2A) = \frac{1}{2}(A+A)$ $A = \frac{1}{2}(A+A'+A-A')$ $A = \underbrace{\frac{1}{2}(A+A')}_{\text{Symmetric}} + \underbrace{\frac{1}{2}(A-A')}_{\text{Skew symmetric}}$ Proof - $\left(\frac{1}{2}(A+A')\right)' = \frac{1}{2}(A'+(A')')$ $= \frac{1}{2}(A'+A)$ $= \frac{1}{2}(A+A')$ st $\therefore \frac{1}{2}(A+A')$ is symmetric $\left(\frac{1}{2}(A-A')\right)' = \frac{1}{2}(A'-(A')')$ $= \frac{1}{2}(A'-A)$ $= \frac{-1}{2}(A-A')$ st $\therefore \frac{1}{2}(A-A')$ is skew symmetric Hence, every square matrix can be represented as sum of symmetric and skew symmetric matrices.	1/2 1 1 1/2	3

20

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$|A| = 2(-4+4) + 3(-6+4) + 5(3-2)$$

$$|A| = 0 - 6 + 5 = -1 \neq 0$$

So, A^{-1} exists

$$\text{Adj } A = \begin{bmatrix} 0 & +2 & 1 \\ -1 & -9 & -5 \\ 2 & +23 & 13 \end{bmatrix}^{T}$$

$$= \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj} A}{|A|} = \frac{-1}{1} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$ 3

$$A^{-1} = \begin{bmatrix} 0 & +1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$ $1\frac{1}{2}$

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ 3 \end{bmatrix}$$

$$A \quad X = B$$

$$X = A^{-1}B$$

using A^{-1} from previous calculation

$$X = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$ $1\frac{1}{2}$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$x = 1, \; y = 2, \; z = 3$$

Extra questions of Set B

1	a (2)	1	1
4	(c) AB+BA is Symmetric Matrix	1	1
7	(c) 4 and not 1	1	1
15	$\underset{1\times 3}{\begin{bmatrix} x & -5 & -1 \end{bmatrix}} \underset{3\times 3}{\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$		
	$\begin{bmatrix} x+0-2 & 0-10+0 & 2x-5-3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$	1/2	
	$\underset{1\times 3}{\begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}} = 0$	1/2	2
	$x^2 - \cancel{2x} - 40 + \cancel{2x} - 8 = 0$	1/2	
	$x^2 = 48$		
	$x = \pm 4\sqrt{3}$	1/2	
19	Given:- A is invertible T.P. :- A is Non Singular. Proof:- Let B be the inverse of A $AB = BA = I$ $\lvert AB \rvert = \lvert I \rvert$ $\lvert A \rvert \, \lvert B \rvert = 1$ $\therefore \lvert A \rvert \neq 0$		

∴ A is non singular $1\frac{1}{2}$ $4\frac{1}{2}$

Given :- A is non singular

T.P.T A is invertible

Proof we know that

$$A(\operatorname{adj} A) = (\operatorname{adj} A)A = |A| I$$

$$A\left(\frac{\operatorname{adj} A}{|A|}\right) = \left(\frac{\operatorname{adj} A}{|A|}\right)A = I$$ 3

$1\frac{1}{2}$

∴ A is invertible

& $A^{-1} = \frac{\operatorname{adj} A}{|A|}$