Half Yearly Exams (2024-25)

Mathematics (Applied) Set: A+B

Marking Scheme / Hints to Solutions

Note:- Any other relevent answer not given here in but given by the students be suitably awarded.

Q. No.	Value points / Key points	Marks allotted to each key point	Total points
1 4(B)	a (-2)	1	1
2 3(B)	b. $(-2, 3]$	1	1
3 1(B)	(a) Symmetric	1	1
4 2(B)	(d) No value of α exist	1	1
5 10(B)	(b) ± 8	1	1
6 6(B)	(b) 20 Sq. unit	1	1
7. 11(B)	(a) $\frac{x-y}{x \log x}$	1	1
8 8(B)	(a) 11	1	1
9 5(B)	(c) 98	1	1
10 9(B)	(a) $\frac{x^2}{2} + \frac{1}{x^2} - \frac{1}{4}$	1	1
11. 14(B)	(b) $\frac{1}{2} \log(ab) \log \frac{b}{a}$	1	1

12 15(B)	(b) 1.5 k.m./h		
13 11(B)	(b) 2 points	1	
14 16(B)	(b) $\frac{1}{\pi}$ units	1	1
15 17(B)	(d) Strictly decreasing	1	1
16 18(B)	(b) 76	1	1
17 17(B)	(a) $<$	1	1
18 12(B)	(b) 7	1	1
19 20(B)	(a) Both A and R are true and R is the correct explanation of A	1	1
20 15(B)	(c) A is true and R is false	1	1
21 24(B)	Let Shorter length $= x$ c.m Then IInd length of piece $= (x+3)$ c.m. Length of IIIrd piece $= 2x$ c.m A.T.Q $x + x + 3 + 2x \leq 91$ $4x \leq 88$ $x \leq 22$ $2x \geq x + 3 + 5$ $x \geq 8$ $8 \leq x \leq 22$ $\therefore$ Length of Shorter piece is 8 or 22 or lying between 8 and 22	1/2 1/2 1/2 1/2	

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{bmatrix}$$ ½

$$A+2B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix} + \begin{bmatrix} -6 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 28 \end{bmatrix}$$ 1

$$= \begin{bmatrix} -4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 37 \end{bmatrix}$$ ½

$$= \text{diag}[-4, 9, 37]$$

2

23 21(B)

$\sqrt{x} + \sqrt{y} = 1$

diff both sides w.r.t x

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$$ 1

$$\frac{1}{\sqrt{y}} \frac{dy}{dx} = \frac{-1}{\sqrt{x}}$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$ ½

$$\frac{dy}{dx} \text{ at } (4,9) = -\sqrt{\frac{9}{4}} = -\frac{3}{2}$$ ½

2

24 22(B)

$$\int x\sqrt{2x+3}\,dx$$

$$\frac{1}{2}\int (2x+3-3)\sqrt{2x+3}\,dx$$ ½

$$\frac{1}{2}\int (2x+3)^{3/2}\,dx - \frac{3}{2}\int (2x+3)^{1/2}\,dx$$ ½

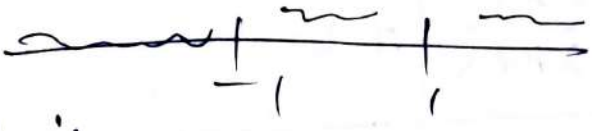
24	$\frac{2}{5}\frac{1}{2}\frac{(2x+3)^{5/2}}{2} - \frac{2}{2}\times\frac{2}{3}\frac{(2x+3)^{3/2}}{2} + C$	1	
	$\frac{1}{10}(2x+3)^{5/2} - \frac{1}{2}(2x+3)^{3/2} + C$		
25	$\int_1^2 e^x \underbrace{\left(\frac{1}{x}}_{f(x)} - \underbrace{\frac{1}{x^2}}_{f'(x)}\right) dx$		
25(B)	$\left[\frac{e^x}{x}\right]_1^2$	1	2
	$\frac{e^2}{2} - \frac{e}{1} = e\left(\frac{e}{2} - 1\right)$	1	
	Section C		
26	$y = 3x^2 + 4$		
28(B)	$\frac{dy}{dx} = 6x =$ Slope of tangent lines	1	
	A.T.Q		
	$6x \times +\frac{1}{6} = +1$	1	3
	$x = 1$		
	$\therefore y = 3+4 = 7$		
	$\therefore$ Req. point $(1, 7)$	1	
27, 30(B)	$f(x) = \frac{x}{1+x^2}$		
	$f'(x) = \frac{(1+x^2)(1) - x(2x)}{(1+x^2)^2}$		
	$= \frac{1+x^2-2x^2}{(1+x^2)^2}$		

for C.V. $f'(x) = 0$

$$\frac{1-x^2}{(1+x^2)^2} = 0$$

$\Rightarrow 1 - x^2 = 0 \Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$ — 1



Case(i) if $x < -1$

$f'(x)$ is -ve — ½

$\therefore f(x)$ is $\downarrow$ in $(-\infty, -1)$

Case(ii) if $-1 < x < 1$

$f'(x)$ is +ve — ½

$\therefore f(x)$ is $\uparrow$ in $(-1, 1)$

Case(iii) if $x > 1$

$f'(x)$ is -ve — ½

$\therefore f(x)$ is $\downarrow$ in $(1, \infty)$

$\therefore (-\infty, -1) \cup (1, \infty) \uparrow$

$(-1, 1) \downarrow$ — ½

3

28 27(B)

$$\int \frac{dx}{4x^2+12x+5}$$

$$\frac{1}{4}\int \frac{dx}{x^2+3x+\frac{5}{4}+\frac{9}{4}-\frac{9}{4}}$$ — ½

$$\frac{1}{4}\int \frac{dx}{(x+3/2)^2-(1)^2}$$ — ½

$$\frac{1}{4} \times \frac{1}{2x1} \log \left| \frac{x+\frac{3}{2}-1}{x+\frac{3}{2}+1} \right| + c$$

$$\frac{1}{8} \log \left| \frac{2x+3-2}{2x+3+2} \right| + c$$

$$\frac{1}{8} \log \left| \frac{2x+1}{2x+5} \right| + c \qquad 1/2$$

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2 29

3(B) $\int \frac{x}{(x-1)^2(x+2)}\,dx$

$$\frac{x}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \qquad 1/2$$

$$\frac{x}{(x-1)^2(x+2)} = \frac{A(x-1)(x+2) + B(x+2) + C(x-1)^2}{(x-1)^2(x+2)}$$

$$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2 \qquad 1/2$$

put $x = 1, -2$

$1 = 3B \Rightarrow B = 1/3$

$-2 = 9C \Rightarrow C = -2/9$

Compare the Co-eff of x^2 3

$0 = A + C \Rightarrow A - \frac{2}{9} = 0 \Rightarrow A = \frac{2}{9}$ 1/2

$$\frac{2}{9}\int \frac{1}{x-1}\,dx + \frac{1}{3}\int \frac{1}{(x-1)^2}\,dx - \frac{2}{9}\int \frac{1}{x+2}\,dx \qquad 1/2$$

$$\frac{2}{9}\log|x-1| + \frac{1}{3}\,\frac{(x-1)^{-1}}{-1} - \frac{2}{9}\log|x+2| + c \qquad 1$$

$$\frac{2}{9}\log|x-1| - \frac{1}{3(x-1)} - \frac{2}{9}\log|x+2| + c$$

Ans

$$\int_{-2}^{2} |x+1|\,dx$$

$$|x+1| = \begin{cases} x+1 & x \geq -1 \\ -(x+1) & x < -1 \end{cases}$$

$$-\int_{-2}^{-1}(x+1)\,dx + \int_{-1}^{2} x+1\,dx \qquad 1$$

$$-\left[\frac{x^2}{2}+x\right]_{-2}^{-1} + \left[\frac{x^2}{2}+x\right]_{-1}^{2} \qquad 1$$

$$-\left[\left(\frac{1}{2}-1\right)-(2-2)\right] + \left[(2+2)-\left(\frac{1}{2}-1\right)\right] \qquad 1/2$$

$$-\left[-\frac{1}{2}\right] + \left[4+\frac{1}{2}\right]$$

$$\frac{1}{2}+\frac{9}{2} = 5 \text{ sq. unit} \qquad 1/2$$

3

31 / 26(B)

$$\int_{10}^{20}(275 - x - 0.3x^2)\,dx \qquad 1$$

$$275x - \frac{x^2}{2} - 0.3\frac{x^3}{3}\Big|_{10}^{20} \qquad 1$$

$$\left[275(20) - \frac{400}{2} - \frac{3}{10}\times\frac{8000}{3}\right] -$$

$$\left[275(10) - \frac{100}{2} - \frac{3}{10}\times\frac{1000}{3}\right]$$

$$5500 - 200 - 800 - 2750 + 50 + 100 \qquad 1$$

$$1900$$

3

Section D

32

33(B) Let the shorter side of $\Delta = x$ c.m

then longer " $= (x+8)$ c.m.

$= (y$ c.m)

$x + y = 23$ $\quad x+8 = y$

$x - y = -8$

$x + y + z = 45$

$x + y + z = 45$

$x - y = -8$

$x + y - 2z = 0$ 1

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ -8 \\ 0 \end{bmatrix}$$ 1½

which is of the form $AX = B$

$|A| = 1(2) - 1(-2) + 1(2)$ ½

$= 2 + 2 + 2 = 6 \neq 0$

$$\text{adj } A = \begin{bmatrix} 2 & +2 & 2 \\ +3 & -3 & 0 \\ 1 & +1 & -2 \end{bmatrix}'$$

$$A^{-1} = \frac{1}{6} \begin{bmatrix} 2 & 3 & 1 \\ 2 & -3 & 1 \\ 2 & 0 & -2 \end{bmatrix}$$ 1½

$X = A^{-1}B$

$$X = \frac{1}{6}\begin{bmatrix} 2 & 3 & 1 \\ 2 & -3 & 1 \\ 2 & 0 & -2 \end{bmatrix}\begin{bmatrix} 45 \\ -8 \\ 0 \end{bmatrix} \quad 1\frac{1}{2}$$

$$= \frac{1}{6}\begin{bmatrix} 90-24 \\ 90+24 \\ 90 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 66 \\ 114 \\ 90 \end{bmatrix} \quad 5$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 19 \\ 15 \end{bmatrix}$$

$\therefore x = 11, \quad y = 19, \quad z = 15$ — 1

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33 (B)

$C(x) = \frac{x^2}{100} + 10x + 40$

$R(x) = px = 20x - \frac{x^2}{400}$ — 1½

$P(x) = R(x) - C(x)$

$P(x) = 20x - \frac{x^2}{400} - \frac{x^2}{100} - 10x - 40$ — 1½

$P(x) = 100x - \frac{x^2}{80} - 40$ — 1½

$\frac{dP}{dx} = 100 - \frac{2x}{80}$ — 1½

for C.V

$\frac{dP}{dx} = 0 \Rightarrow 100 = \frac{2x}{80}40$ — 1½

$x = 4000$

$\frac{d^2p}{dx^2} = -\frac{2}{80} < 0$ when $x = 4000$

$x = 4000$ is the point of maximum $\frac{1}{2}$

price per unit $= 200 - \frac{10}{400} \times 4000$ $\frac{1}{2}$

$= ₹ 180$

Profit $= 100(4000) - 4000 \times \frac{50}{80} - 40$ $\frac{1}{2}$ 5

$= 400000 - 200000 - 40$

$= 200000 - 40$

$= ₹ 199960$ $\frac{1}{2}$

34 32(B) The given demand function is

$p = 274 - x^2$

$M.C. = 4 + 3x$

$R = px = 274x - x^3$ $\frac{1}{2}$

$MR = \frac{dR}{dx} = 274 - 3x^2$ $\frac{1}{2}$

Under Monopoly $MR = MC$ $\frac{1}{2}$

$274 - 3x^2 = 4 + 3x$

$3x^2 + 3x - 270 = 0$

$x^2 + x - 90 = 0$

$(x+10)(x-9) = 0$

$x = -10$ or $x = 9$

$x \neq -10$ $\therefore x = 9$ $1\frac{1}{2}$

when $x_0 = 9$ $p_0 = 274 - (9)^2 = 193$

$$C.S. = \int_0^b p\,dx - p_0 x_0 \quad \cdots (1)$$

1

$$= \int_0^9 (274 - x^2)\,dx - 193 \times 9$$

$$\left[274x - \frac{x^3}{3}\right]_0^9 - 1737$$

5

$$274 \times 9 - \frac{9^3}{3} - 1737 = 486$$

1

35

34(B)

$$x^p y^q = (x+y)^{p+q}$$

Taking log both sides

$$p \log x + q \log y = (p+q) \log (x+y)$$

1

diff both sides w.r.t. x

$$p \cdot \frac{1}{x} + q \cdot \frac{1}{y}\frac{dy}{dx} = (p+q)\left[\frac{1}{x} + \frac{1}{y}\frac{dy}{dx}\right]$$

1

$$\frac{p}{x} - \frac{p+q}{x} = \left[\frac{p+q}{y} - \frac{q}{y}\right]\frac{dy}{dx}$$

1½

5

$$\frac{px - px - qx}{x} = \left[\frac{py + qy - qy}{y}\right]\frac{dy}{dx}$$

1

$$-\frac{q}{p} = \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = 0$$

1/2

Section E

36 Let A(-1,2) B(2,3) C(6,1)

ar of $\Delta ABC = \frac{1}{2}\begin{vmatrix} -1 & 2 & 1 \\ 2 & 3 & 1 \\ 6 & 1 & 1 \end{vmatrix}$

37(B) (i)

$= \frac{1}{2}|-1(2) - 2(-4) + 1(-16)|$

$= \frac{1}{2}|-2 + 8 - 16|$

$= \frac{1}{2}|-10| = \frac{1}{2} \times 10 = 5$ sq. unit $1\frac{1}{2}$

(ii) Eq. of line AB :-

$\frac{1}{2}\begin{vmatrix} 2 & 1 & 1 \\ 3 & 5 & 1 \\ x & y & 1 \end{vmatrix} = 0$ 4

$\frac{1}{2}|2(5-y) - 1(3-x) + 1(3y-5x)| = 0$

$10 - 2y - 3 + x + 3y - 5x = 0$

$-4x + y + 7 = 0$ $1\frac{1}{2}$

$4x - y - 7 = 0$

(iii) $\frac{1}{2}\begin{vmatrix} 2 & -3 & 1 \\ k & -1 & 1 \\ 0 & 4 & 1 \end{vmatrix} = 0$

$2(-5) + 3(k) + 1(4k) = 0$

$-10 + 3k + 4k = 0$ 1

$7k = 10$

$k = \frac{10}{7}$

$C(x)$ be the maintenance function then

$$C(x) = 5000000 + 160x - 0.04x^2$$ 1

we have

$$C(x) = 5000000 + 160x - 0.04x^2$$

$$C'(x) = 160 - 0.08x$$

for c.v $C'(x) = 0$

$$160 = 0.08x$$

$$x = 2000$$

$$C''(x) = -0.08 < 0$$ 2

$\therefore x = 2000$ is the point of maxima.

The complex must have 4500 apartment to minimize the maintenance cost. 1

or

The total maintenance cost for 4500 apartments

$$= 5000000 + 160(4500) - 0.04(4500)^2$$

$$= 5000000 + 720000 - 810000$$

$$= 4910000$$

Minimum maintenance cost is ₹ 4910000 1

4

8 (i) (8B) Part of tank filled by pipe A and B in 1 hour $= \frac{1}{6}$

" " " " " B + C $= \frac{1}{10}$

" " " " " A + C $= \frac{2}{15}$

Part of tank filled by A, B, C in 2 hours $= \frac{1}{6} + \frac{1}{10} + \frac{2}{15}$

$= \frac{24}{60}$

Part of tank filled by A, B, C in 1 hours $= \frac{24}{60} \times \frac{1}{2} = \frac{1}{5}$ 2 2

$\therefore$ Time taken by A, B, C $= 5$ h

(ii) Part of tank filled by A, B, C in 1 hour $= \frac{1}{5}$

Part of tank filled by B & C in 1 hour $= \frac{1}{10}$

Part of tank filled by A in 1h $= \frac{1}{5} - \frac{1}{10}$ 1 1

$= \frac{2-1}{10}$

$= \frac{1}{10}$

$\therefore$ A Separately can fill the tank in 10h.

(iii) Part of tank filled by B in 1h

$= \frac{1}{5} - \frac{2}{15}$

$= \frac{3-2}{15} = \frac{1}{15}$

$\therefore$ B Separately can fill the tank in 15 h.

Part of tank filled ...

OR

Part of tank filled by tank C in 1 h

$= \frac{1}{5} - \frac{1}{6}$

$= \frac{6-5}{30} = \frac{1}{30}$ 1 1

C Separately can fill the tank in 30 h