



**O.S.D.A.V. PUBLIC SCHOOL, KAITHAL**  
**HALF YEARLY EXAMS (2024-25)**  
**CLASS-XII**  
**SUBJECT : MATHEMATICS**

**Set B**

**Time allowed: 3 Hrs.**

**Maximum Marks: 80**

**General Instructions:-**

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 very short answer (VSA)-type questions of 2 marks each.
4. Section C has 6 short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 long answer (LA) type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assertion of 4 marks each with sub-parts.

**Section-A**

(All questions are compulsory. No internal choice is provided in this section)

1. The value of  $\sin^{-1}\left(\frac{-1}{2}\right) - \sin^{-1}(-1)$ :  
a.  $\frac{\pi}{6}$                       b.  $\frac{5\pi}{6}$                       c.  $\frac{-\pi}{6}$                       d.  $\frac{\pi}{3}$
2. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function defined by  $f(x) = x^3 + 4$  then  $f$  is:  
a. injective                      b. surjective                      c. bijective                      d. none of these
3. If  $|A| = |kA|$ , where  $A$  is a square matrix of order 2, then sum of all possible of  $k$  is:  
a. 1                      b. -1                      c. 2                      d. 0
4. The domain of the function defined by  $\sin^{-1}(\sqrt{x-1})$  is:  
a.  $[1, 2]$                       b.  $[-1, 1]$                       c.  $[0, 1]$                       d.  $[0, 2]$
5. If  $x = 2$  at  $y = at^2$  then  $\frac{d^2y}{dx^2}$  at  $t = \frac{1}{2}$  is:  
a.  $\frac{1}{2a}$                       b. 1                      c.  $2a$                       d.  $-2a$
6. Write the order of the product matrix:  $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$  '  
a.  $1 \times 1$                       b.  $3 \times 3$                       c.  $1 \times 3$                       d.  $3 \times 1$
7. If  $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$  and  $A^2 - KA - 5I = 0$ , then value of  $k$  is:  
a. 3                      b. 7                      c. 5                      d. 9
8. Let  $A = \{1, 2, 3\}$  and  $R = \{(1, 2), (2, 3)\}$  be a relation in  $A$ . Then the minimum number of ordered pairs may be added, so that  $R$  becomes an equivalence relation is:  
a. 7                      b. 5                      c. 1                      d. 4
9. If  $y = (\sqrt{\tan x})$  then  $\frac{dy}{dx}$  at  $x = \frac{\pi}{4}$  is:  
a. 1                      b. 0                      c.  $\frac{1}{2}$                       d. none of these
10.  $A$  is a square matrix of order 2 and  $|A| = 7$ , then find the value of  $|2AA'|$

11.  $\int e^{\log \sin x} dx$  is equal to:  
 a.  $\sin x + c$                       b.  $\cos x + c$                       c.  $-\cos x + c$                       d.  $-\sin x + c$
12. If  $f(x) = \begin{cases} ax + 3 & 0 < x \leq 1 \\ 2x^2 - x & 1 < x < 2 \end{cases}$  is continuous on  $(0, 2)$  then the value of  $a$  is:  
 a. 2                      b. 1                      c. -1                      d. -2
13.  $\int_0^{\pi^2/4} \frac{\sin\sqrt{x}}{\sqrt{x}} dx$  is equal to:  
 a. 2                      b. 1                      c.  $\pi$                       d.  $\pi/2$
14. If the sides of square are decreasing at the rate of 1.5c.m./sec. the rate of decrease of its perimeter is:  
 a. 1.5cm/sec                      b. 6cm/sec                      c. 3cm/sec.                      d. 2.25cm/sec.
15. The function  $f(x) = ax + b$  is strictly decreasing for all  $x \in \mathbb{R}$  if:  
 a.  $a = 0$                       b.  $a < 0$                       c.  $a > 0$                       d.  $a = 1$
16. If  $x = a^{\sin^{-1}t}$  and  $y = a^{\cos^{-1}t}$  then  $\frac{dy}{dx} =$   
 a. 0                      b. 1                      c.  $x$                       d.  $y$
17. If  $A$  is a singular matrix, then  $A$  (adj  $A$ ) is:  
 a. Null Matrix                      b. Scalar Matrix                      c. Identity matrix                      d. None of these
18. If  $A$  is a skew symmetric matrix of order 3, then the value of  $|A|$  is:  
 a. 0                      b. 3                      c. 9                      d. 27

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer and of the following choices.

- a. Both (A) and (R) are true and (R) is the correct explanation of (A).  
 b. Both (A) and (R) are true and (R) is not the correct explanation of (A).  
 c. (A) is true but (R) is false.  
 d. (A) is false but (R) is true.
- e. 20. Assertion (A): The minimum value of  $f(x) = x^2 + 2bx + c$  is  $c - b^2$   
 f. Reason (R) :  $f'(-b) = 0$
19. Assertion (A) : Principal value of  $\sin^{-1}(\frac{1}{\sqrt{2}}) = \pi/4$   
 Reason (R) : Principal value of  $\cot^{-1}(\frac{1}{\sqrt{3}}) = 2\pi/3$
20. Assertion (A): The minimum value of  $f(x) = x^2 + 2bx + c$  is  $c - b^2$   
 Reason (R) :  $f'(-b) = 0$

### Section B

21.  $y = \cos^{-1}(\frac{1-x^2}{1+x^2})$ ,  $0 < x < 1$  find  $\frac{dy}{dx}$
22. Evaluate :  $\int \frac{1}{\sqrt{x^2+2x+4}} dx$
23. If  $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$  and  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  find  $k$  so that  $A^2 = KA - 2I$
24. Show that the function given by  $f(x) = \frac{\log x}{x}$  has maximum at  $x = e$

25. Evaluate  $\int \frac{\cos 2x}{(\cos x + \sin x)^2} dx$

**Section C**

26. If  $y = (\tan^{-1} x)^2$  show that  $(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2$

27. Evaluate  $\int \frac{2x+1}{(x+1)^2(x-1)} dx$

28. Show that  $y = \log(1+x) - \frac{2x}{2+x}$ ,  $x > -1$  is an increasing function of  $x$  throughout its domain.

29. Evaluate  $\int e^x \left( \frac{1+\sin x}{1+\cos x} \right) dx$

30. Solve :  $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

31. Find  $x$  if:

$$\begin{bmatrix} x & -5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

**Section – D**

32. Differentiate the following w.r.t.  $x$   $\sin^{-1} \left( \frac{2^{x+1}}{1+4^x} \right)$

33. If  $A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$  find  $A^{-1}$

Hence solve the system of equations:

$$3x + 3y + 2z = 1$$

$$x + 2y = 4$$

$$2x - 3y - z = 5$$

34.  $\int_0^{\pi/2} (2 \log \sin x - \log \sin 2x) dx$

35. A Relation  $R$  is defined on  $N \times N$  as:

$$(a, b) R (c, d) \Leftrightarrow a - c = b - d.$$

Show that  $R$  is an equivalence relation.

**Section E**

36. A class teacher wants to make different groups of students so that they can be given different tasks of enlightening other about the effects of poor AQI (Air Quality Index) level. Students are making groups with friends but the teacher said not like this, we will make a group of students with roll number in such a way that the difference of roll number is divisible by 3.



Based on the above information, answer the following questions:

- i. Name the properties which whole group should satisfy to get divided into different groups (equivalence classes).
- ii. Provide the relation for the roll number of students in the group of student with roll number 5.
- iii. Which roll number students will be in the group of students with roll number 5 if there are 30 students in the class?

or

Which roll number students will be in the group of student with roll number 2, if there are 20 students in the class?

37. A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic metres. Cost of land is Rs. 500 per square metre. The cost of digging increases with the depth and cost for the whole pool is Rs. 4000 (depth)<sup>2</sup>.



Suppose the side of the square plot is  $x$  metres and depth is  $h$  metres. On the basis of the above information, answer the following questions:

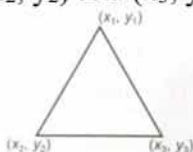
- Write cost  $C(h)$  as a function in terms of  $h$ . 1
- Find critical point. 1
- (a) Use second derivative test to find the value of  $h$  for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool? 2

or

- (b) Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also find relation between  $x$  and  $h$  for minimum cost. 2

38. Area of a triangle whose vertices are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  is given by the determinant

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$



Since, area is a positive quantity, so we always take the absolute value of the determinant. Also, the area of the triangle formed by three collinear points is zero. Based on the above information, answer the following questions:

- Find the area of the triangle whose vertices are  $(-2, 6)$ ,  $(3, -6)$  and  $(1, 5)$ .
- Using determinants, find the equation of the line joining the points  $A(1, 3)$  and  $B(0, 0)$ .
- If the points  $(2, -3)$ ,  $(k, -1)$  and  $(0, 4)$  are collinear, then find the value of  $k$ .

or

If the area of a  $\triangle ABC$  with vertices  $A(1, 3)$ ,  $B(0, 0)$  and  $C(k, 0)$  is 6 sq. units, then find the value of  $k$ .

# Half Yearly Exams (2024-25)

## Mathematics (Core)

Set A+B

### Marking Scheme/Hints to Solutions

Note:- Any other relevant answer not given here in but given by the students be suitably awarded.

Q.No.	Value points / Key points	Marks allotted to each key point	Total points
1 8(B)	(a) 7	1	1
2 2(B)	(c) Bijective	1	1
3 1(B)	(d) $\pi/3$	1	1
4 4(B)	(a) $[1, 2]$	1	1
5 6(B)	(b) $3 \times 3$	1	1
6 7(B)	(b) 7	1	1
7 3(B)	(d) 0	1	1
8 10(B)	(a) 196	1	1
9 3(B)	(a) 1	1	1
10 5(B)	(a) $\frac{1}{2a}$	1	1
11 12(B)	(d) -2	1	1
12 14(B)	(b) 6 C.m/Sec.	1	1

13 15(B)	(b) $a < 0$		
14 11(B)	(c) $-\cos x + C$		1
15 13(B)	(a) 2		1
16 7(B)	(a) Null Matrix		1 1
17 (B)	(a) $\infty$		1 1
18 16(B)	(d) $-\frac{y}{x}$		1 1
19 20(B)	(b) Both A and R are true and R is not the correct explanation of A.		1 1
30 20 19(B)	(b) Both A and R are true and R is not the correct explanation of A.		1 1
21 22(B)	$\int \frac{1}{\sqrt{x^2+2x+4}} dx$ $\int \frac{1}{\sqrt{x^2+2x+4+1-1}} du$ $\int \frac{1}{\sqrt{(x+1)^2+(\sqrt{3})^2}} dx.$ $\log  (x+1) + \sqrt{x^2+2x+4}  + C$		1/2 1/2 1
22 21(B)	$y = \cos^{-1} \left( \frac{1-x^2}{1+x^2} \right)$ <p style="text-align: center;">put <math>x = \tan \theta</math></p> $y = \cos^{-1} \left( \frac{1-\tan^2 \theta}{1+\tan^2 \theta} \right)$		1/2

$$y = \cos^{-1}(\cos 2\theta)$$

$$y = 2\theta$$

$$y = 2 \tan^{-1} x$$

$$\frac{dy}{dx} = \frac{2}{1+x^2}$$

1/2

1/2

2

1/2

23

24(B)

$$f(x) = \frac{\log x}{x}$$

$$f'(x) = \frac{x(\frac{1}{x}) - \log x \times 1}{x^2}$$

$$f'(x) = \frac{1 - \log x}{x^2}$$

for c.v.

$$f'(x) = 0 \Rightarrow 1 - \log x = 0$$

$$\log x = 1$$

$$x = e^1 = e$$

$$f''(x) = \frac{x^2[-\frac{1}{x}] - [1 - \log x][2x]}{x^4}$$

$$= \frac{-x - 2x + 2x \log x}{x^4}$$

$$= \frac{-3x + 2x \log x}{x^4}$$

$$f''(e) = \frac{-3e + 2e \log e}{e^4} = \frac{-e}{e^4}$$

∴ e is the point of Maxima =  $-\frac{1}{e^3} < 0$

1/2

1/2

2

1/2

1/2

1/2

24

25(B)

$$\int \frac{\cos 2x}{(\cos x + \sin x)^2} dx$$

$$\int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2} dx$$

$$\int \frac{(\cancel{\cos x + \sin x})(\cos x - \sin x)}{(\cos x + \sin x)^2} dx$$

$$\log |\cos x + \sin x| + C$$

1/2

1/2

2

1

25

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

$$A^2 = kA - 2I$$

$$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$1 = 3k - 2$$

$$3 = 3k$$

$$k = 1$$

$$-2 = -2k$$

$$k = 1 \text{ --- Ans}$$

1

2

1

26

31(B)

### Section C

$$\begin{bmatrix} x & -5 & 1 \end{bmatrix}_{1 \times 3} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}_{3 \times 3} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$



$$\begin{bmatrix} x+2 & -10 & 2x-5+3 \end{bmatrix}_{1 \times 3} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0 \quad 1$$

$$x^2 + 2x - 40 + 2x - 2 = 0 \quad 1$$

$$x^2 + 4x - 42 = 0$$

$$x = \frac{-4 \pm \sqrt{16 - 4 \times 1 \times -42}}{2}$$

$$x = \frac{-4 \pm \sqrt{16 + 168}}{2}$$

$$x = \frac{-4 \pm 2\sqrt{46}}{2} = -2 \pm \sqrt{46} \quad 1$$

3

27

27(B)

$$\int \frac{2x+1}{(x+1)^2(x-1)} dx$$

$$\frac{2x+1}{(x+1)^2(x-1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \quad 1/2$$

$$\frac{2x+1}{(x+1)^2(x-1)} = \frac{A(x+1)^2 + B(x+1)(x-1) + C(x-1)}{(x+1)^2(x-1)}$$

put  $x=1, -1$  Compare the  
 $3 = 4A \Rightarrow \boxed{\frac{3}{4} = A}$  ( $x^2$ ) Coefft

$+1 = +2C \Rightarrow \boxed{\frac{1}{2} = C}$   
 $0 = A+B$   
 $0 = \frac{3}{4} + B$   
 $B = -\frac{3}{4}$

1

$$\frac{3}{4} \int \frac{1}{x-1} dx - \frac{3}{4} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{1}{(x+1)^2} dx$$

$$\frac{3}{4} [\log|x-1| - \log|x+1|] + \frac{1}{2} \frac{(x+2)^{-1}}{-1} + C \quad 1$$

$$\frac{3}{4} \log \left| \frac{x-1}{x+1} \right| - \frac{1}{2(x+2)} + C \text{ --- Ans}$$

1/2

28  $y = (\tan^{-1} x)^2$

1

28(b)  $y_1 = \frac{2 \tan^{-1} x}{1+x^2}$

1/2

$$(1+x^2)y_1 = 2 \tan^{-1} x$$

$$(1+x^2)y_2 + y_1(2x) = \frac{2}{1+x^2}$$

1/2

$$(1+x^2)^2 y_2 + 2x(1+x^2)y_1 = 2$$

3

29  $y = \log(1+x) - \frac{2x}{2+x}, x > -1$

28(b)

$$\frac{dy}{dx} = \frac{1}{1+x} - \left[ \frac{(2+x)(2) - 2x(1)}{(2+x)^2} \right]$$

1

$$= \frac{1}{1+x} - \left[ \frac{4+2x-2x}{(2+x)^2} \right]$$

$$= \frac{4+x^2+4x-4-4x}{(1+x)(2+x)^2}$$

$$= \frac{x^2}{(1+x)(2+x)^2}$$

1

$$x^2 \wedge (2+x)^2 > 0$$

$\therefore \frac{dy}{dx}$  is +ve  $\forall x > -1$

1/2

$\therefore f(x)$  is increasing for  $x > -1$

$$\int e^x \left( \frac{1 + \sin x}{1 + \cos x} \right) dx$$

$$\int e^x \left( \frac{1}{1 + \cos x} + \frac{\sin x}{1 + \cos x} \right) dx$$

$$\int e^x \left( \frac{1}{2 \cos^2 \frac{x}{2}} + \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right) dx$$

$$\int e^x \left( \frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right) dx$$

$$e^x \tan \frac{x}{2} + C \text{ --- Ans}$$

1/2

1/2

1

1

1/2

29(b)

31  
30(B)

$$2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$$

$$\tan(2 \tan^{-1}(\cos x)) = \frac{2}{\sin x} \quad // 2$$

$$\frac{2 \tan(\tan^{-1}(\cos x))}{1 - \tan^2(\tan^{-1}(\cos x))} = \frac{2}{\sin x}$$

$$\frac{2 \cos x}{1 - \cos^2 x} = \frac{2}{\sin x} \quad // 2$$

$$\frac{\cos x}{1 - \cos^2 x} = \frac{1}{\sin x} \quad // 2$$

$$\frac{\cos x}{\sin^2 x} = \frac{1}{\sin x}$$

$$\sin x (\cos x - \sin^2 x) = 0 \quad // 2$$

$$\sin x (\cos x - \sin x) = 0$$

Reject  $\sin x = 0$  or  $\sin x = \cos x$   
 $x = 0$  or  $x = \pi/4$  — Ans  $// 2$

3

Section D

32  
33(b)

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$$

$$|A| = 3(-2) - 1(-3 + 6) + 2(-4) \\ = -6 - 3 - 8 = -17 \neq 0$$

$$\text{adj}A = \begin{bmatrix} -2 & -3 & -4 \\ +1 & -7 & +2 \\ -7 & +15 & 3 \end{bmatrix}'$$

$$= \begin{bmatrix} -2 & 1 & -7 \\ -3 & -7 & 15 \\ -4 & 2 & 3 \end{bmatrix}$$

$$A^{-1} = -\frac{1}{17} \begin{bmatrix} -2 & 1 & -7 \\ -3 & -7 & 15 \\ -4 & 2 & 3 \end{bmatrix}$$

$$3x + 3y + 2z = 1$$

$$x + 2y = 4$$

$$2x - 3y - z = 5$$

$$\begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 0 \\ 2 & -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$$

$$A' X = B$$

$$X = (A')^{-1} B = (A^{-1})' B$$

1/2

1

1/2

1/2

1

$$X = -\frac{1}{17} \begin{bmatrix} -2 & -3 & -4 \\ 1 & -7 & 2 \\ -7 & 15 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$$

$$= -\frac{1}{17} \begin{bmatrix} -2-12-20 \\ -28+10 \\ -7+60+15 \end{bmatrix} = -\frac{1}{17} \begin{bmatrix} -34 \\ -17 \\ 68 \end{bmatrix}$$

$$x = 2 \quad y = \frac{17}{17} = +1 \quad z = -4$$

$$x = 2, \quad y = +1, \quad z = -4$$

1

5

 $\frac{1}{2}$ 

33  
33  
(b)

$(a, b) R (a, b)$

$$\because a - a = b - b \quad \forall (a, b) \in N \times N$$

$$i.e. 0 = 0$$

$\therefore R$  is Reflexive

if  $(a, b) R (c, d)$

$$\therefore a - c = b - d$$

$$\therefore -(a - c) = -(b - d) \quad \forall (a, b), (c, d)$$

$$\therefore c - a = d - b \quad \in N \times N$$

$$\therefore (c, d) R (a, b)$$

$\therefore R$  is Symmetric

if  $(a, b) R (c, d)$  and  $(c, d) R (e, f)$

$$\therefore a - c = b - d \text{ --- (1)} \quad \therefore c - e = d - f \text{ --- (2)}$$

$$\therefore a - \cancel{c} + \cancel{c} - e = b - \cancel{d} + \cancel{d} - f$$

 $\frac{1}{2}$  $\frac{1}{2}$

$$\therefore a - c = b - d$$

$$\therefore (a, b) R (c, d) \quad \forall (a, b), (c, d), (e, f) \in N \times N$$

$\therefore R$  is transitive

$\therefore R$  is an equivalence relation.

1/2

1/2

34

34(b)

$$\text{let } y = \sin^{-1} \left( \frac{2^{x+1}}{1+4^x} \right)$$

$$\text{put } 2^x = \tan \theta$$

$$y = \sin^{-1} \left( \frac{2 \tan \theta}{1 + \tan^2 \theta} \right)$$

$$y = \sin^{-1} (\sin 2\theta)$$

$$y = 2\theta$$

$$y = 2 \tan^{-1} 2^x$$

$$\frac{dy}{dx} = \frac{2}{1+(2^x)^2} 2^x \log 2$$

$$\frac{dy}{dx} = \frac{2^{x+1} \log 2}{1+4^x} \quad \text{--- Ans}$$

1/2

1/2 + 1/2

1/2

1/2

1/2

5

1/2

1/2

35

34(b)

$$I = \int_0^{\pi/2} (2 \log \sin 2x - \log \sin 2x) dx$$

$$I = \int_0^{\pi/2} (\log \sin^2 x - \log \sin 2x) dx$$

1/2

$$I = \int_0^{\pi/2} \log \frac{\sin^2 x}{\sin 2x} dx$$

1/2

$$I = \int_0^{\pi/2} \log \frac{\sin^2 x}{2 \sin x \cos x} dx$$

$$I = \int_0^{\pi/2} \log \frac{\tan x}{2} dx$$

1/2

$$I = \int_0^{\pi/2} \log \tan x dx - \int_0^{\pi/2} \log 2 dx$$

1/2

$$I = I_1 - [x \log 2]_0^{\pi/2}$$

$$I = 0 - \frac{\pi}{2} \log 2 = -\frac{\pi}{2} \log 2 - \text{Ans}$$

1/2

5

$$I_1 = \int_0^{\pi/2} \log \tan x dx \quad \text{--- (1)}$$

$x \rightarrow \pi/2 - x$

$$I_1 = \int_0^{\pi/2} \log \tan(\frac{\pi}{2} - x) dx$$

2 1/2  
for I<sub>1</sub>

$$I_1 = \int_0^{\pi/2} \log \cot x dx \quad \text{--- (2)}$$

$$2I_1 = \int_0^{\pi/2} \log \tan x \cdot \cot x dx$$

$$2I_1 = \int_0^{\pi/2} \log 1 dx \Rightarrow 2I_1 = 0$$

$I_1 = 0$



Section E

Capacity = area  $\times$  depth =  $\pi r^2 h$   
 $= 250$

$\Rightarrow r^2 = \frac{250}{h}$

(i)

Cost (C) =  $500r^2 + 4000h^2$

27(B)

$C = 2500\left(\frac{250}{h}\right) + 4000h^2$

$\frac{dC}{dh} = -\frac{125000}{h^2} + 8000h$

for c.v

$\frac{dC}{dh} = 0$

$-\frac{125000}{h^2} = -8000h$

(ii)

$\frac{125}{8} = h^3 \Rightarrow h = \frac{5}{2}$

$\frac{d^2C}{dh^2} = 125000 \times \frac{2}{h^3} + 8000$

is +ve when  $h = \frac{5}{2}$

$\therefore h = \frac{5}{2}$  is the point of minimum

Minimum cost =  $\frac{25000}{5} \times 2 + 4000\left(\frac{25}{4}\right)$

$= 50000 + 25000$

$= ₹ 75000$

or

$h$	$h < \frac{5}{2}$	$h > \frac{5}{2}$
$\frac{dC}{dh}$	-ve	+ve

4

1

2

$$x^2 = \frac{250}{h} \Rightarrow x^2 = \frac{250}{\frac{5}{2}} = 100$$

$$\therefore x = 10m$$

$$\therefore x = 4h$$

2

37. (i) The properties are reflexive, symmetric and transitive.

1

(ii) Let  $x$  be the roll number of student who is in the group.

$$\therefore A.T.\phi$$

$$x - 5 = 3d$$

(iii) Roll number of students can be calculated from

$$x - 5 = 3d$$

if  $d = 0, 1, 2, 3, 4, \dots$

$$x = 5, 8, 11, 14, 17, 20, \dots$$

So roll number of students in the required group

$$= [5] = \{2, 5, 8, 11, 14, 17, 20, 23, 26, 29\}$$

2

or

$$x - 2 = 3d$$

$$\text{Group } [2] = \{2, 5, 8, 11, 14, 17, 20\}$$

2

$$\text{Area of } \Delta = \frac{1}{2} \begin{vmatrix} -2 & 6 & 1 \\ 3 & -6 & 1 \\ 1 & 5 & 1 \end{vmatrix}$$

$$\frac{1}{2} [-2(-6-5) - 6(3-1) + 1(5+6)]$$

$$\frac{1}{2} \times 31 = 15.589 \text{ unit}$$

1

38(B)

(ii) Let  $P(x, y)$  be any point lying on the line joining the points  $A(1, 3)$  and  $B(0, 0)$ . So points  $A, P, B$  are collinear.

$$\text{So } \frac{1}{2} \begin{vmatrix} 1 & 3 & 1 \\ x & y & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$$1(y) - 3(x) + 1(0-0) = 0$$

$$y = 3x$$

(iii) Since the points  $A(2, -3), B(k, -1), C(0, 4)$  are collinear then

$$\frac{1}{2} \begin{vmatrix} 2 & -3 & 1 \\ k & -1 & 1 \\ 0 & 4 & 1 \end{vmatrix} = 0$$

$$2(-1-4) + 3(k-0) + 1(4k) = 0$$

$$-7k = 10$$

$$k = \frac{10}{-7}$$

1

4

2

or

Area of  $\Delta ABC = 6$  sq. units

$$\frac{1}{2} \begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 6$$

$$1(0-0) - 3(-k) + 1(0) = \pm 6 \times 2$$

$$3k = \pm 6 \times 2$$

$$k = \frac{\pm 12}{3} = \pm 4$$

2