Half Yearly Exams	Mathematics (Core)	2024-25
	Marking Scheme / Hints to Solutions	

16/09/24

Note:- Any other relevant answer not given here in but given by the students, be suitably awarded.

Q. No.	Value points / Key points	Marks allotted to each Key point	Total points
1 / 5(B)	(a) one-one but not onto	1	1
2 / 4(B)	(c) transitive if (1,1), (2,2) is added.	1	1
3 / 6(B)	(a) $\pi/8$	1	1
4	(d) $[1, 2]$	1	1
5	(d) 25	1	1
6	(b) 9	1	1
7 / 12(B)	(c) 2	1	1
8 / 13(B)	(c) ± 2	1	1
9 / 15(B)	(a) 1000	1	1
10	(c) 1/9	1	1
11 / 2(B)	(b) $\frac{2x}{1+x^4}$	1	1

12	(b) $2e^{x}$		
13 3(B)	(d) neither maximum nor minimum	1	
14 4(B)	(c) $900\ cm^3/sec$	1	
15 17(B)	(a) 2	1	1
16	(b) $f'(x) > 0\ \forall\ x \in (a, b)$	1	1
17 18(B)	(b) $-\cot x - \tan x + C$	1	1
18 16(B)	(c) $\pi/2$	1	1
19 20(B)	(c) A is true but R is false	1	1
20 19(B)	(a) Both A and R are true and R is the correct explanation of A	1	1
21 22(B)	$\begin{bmatrix} x & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x \\ 3 \end{bmatrix} = 0$		
	$\begin{bmatrix} x-2 & -3 \end{bmatrix}\begin{bmatrix} x \\ 3 \end{bmatrix} = 0$	1/2	
	$x^2 - 2x - 9 = 0$	1/2	2
	$x = \dfrac{2 \pm \sqrt{40}}{2} = \dfrac{2 \pm 2\sqrt{10}}{2}$	1/2	
	$x = 1 \pm \sqrt{10}$	1/2	
22 24(B)	$f(x) = \begin{cases} \dfrac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & 1 \le x < 0 \\ \dfrac{2x+1}{x-2} & 0 \le x \le 1 \end{cases}$		

L.H.L

$$\lim_{x \to 0^-} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} \times \frac{\sqrt{1+kx} + \sqrt{1+kx}}{\sqrt{1+kx} + \sqrt{1+kx}}$$

$$\lim_{x \to 0^-} \frac{\cancel{1}+kx - \cancel{1}+kx}{x\left[\sqrt{1+kx} + \sqrt{1-kx}\right]}$$ 1/2

$$\lim_{x \to 0^-} \frac{2k\cancel{x}}{\cancel{x}\left[\sqrt{1+kx} + \sqrt{1-kx}\right]}$$

$$\frac{2k}{\sqrt{1} + \sqrt{1}} = \frac{2k}{2} = k$$ 1/2

2

R.H.L.

$$\lim_{x \to 0^+} \frac{2x+1}{x-2}$$

$x = 0 + h$

$$\lim_{h \to 0} \frac{2(h)+1}{h-2}$$ 1/2

$$-\frac{1}{2}$$

L.H.L = R.H.L [∵ f(x) is cts at x = 0] 1/2

$$k = -\frac{1}{2}$$

23 $f(x) = 5 + 36x + 3x^2 - 2x^3$

$f'(x) = 36 + 6x - 6x^2$ 1/2

	$f'(x) = -6(x-3)(x+2)$ for c.r $f'(x)=0$ $x = 3, -2$ $f'(x)$ is >0 in $(-2,3)$ $\therefore$ $f(x)$ is st $\uparrow$ in $(-2,3)$	1/2 1/2 1/2	2
24	$\int \frac{dx}{\sqrt{3-2x-x^2}}$ $\int \frac{dx}{\sqrt{(2)^2-(x+1)^2}} dx$ $\sin^{-1}\frac{x+1}{2} + c$	1 1	2
25	$\int \frac{(x-4)e^x}{(x-2)^3} dx$ $\int \frac{(x-2-\cancel{2})e^x}{(x-2)^3} dx$ $\int \left[\frac{1}{(x-2)^2} - \frac{2}{(x-2)^3}\right] e^x dx$ $e^x \cdot \frac{1}{(x-2)^2} + C$ — Ans	1/2 1/2 1	2
26 27(B)	$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$ $\tan^{-1}\left(\frac{\sin(\frac{\pi}{2}-x)}{1-\cos(\frac{\pi}{2}-x)}\right)$	1/2	

	$\tan^{-1}\left(\dfrac{\cancel{2}\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{\cancel{2}\sin^{2}\left(\frac{\pi}{4}-x/2\right)}\right)$	1	
	$\tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$	1/2	3
	$\tan^{-1}\left(\tan\left(\frac{\pi}{2}-\frac{\pi}{4}+\frac{x}{2}\right)\right)$	1/2	
	$\frac{\pi}{4}+\frac{x}{2}$ — Ans	1/2	
26	or		
	$\tan^{-1}(1)+\cos^{-1}\left(\frac{-1}{2}\right)+\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$		
	$\cancel{\frac{\pi}{4}}+\pi-\frac{\pi}{3}-\cancel{\frac{\pi}{4}}$	$\frac{1}{2}+1+1$	3
	$2\pi/3$	1/2	
27 30(B)	$f(x)=(\sin x)^{\sin x}+a^{2x}$		
	let $v=(\sin x)^{\sin x}$	1/2	
	$\log v=\sin x\log\sin x$		
	$\frac{1}{v}\frac{dv}{dx}=\cancel{\sin x}\times\frac{\cos x}{\cancel{\sin x}}+\log\sin x\,[\cos x]$		
	$\frac{dv}{dx}=(\sin x)^{\sin x}\left[\cos x(1+\log\sin x)\right]$	$1\frac{1}{2}$	3
	$f'(x)=(\sin x)^{\sin x}\cos x\left[1+\log\sin x\right]+2a^{2x}\log a$	1	

Or

27 $y = e^{\sec^2 x} + 3\sin^{-1}(x^2)$

$\frac{dy}{dx} = e^{\sec^2 x} \times 2\sec x(\sec x \tan x) + \frac{3 \times 2x}{\sqrt{1-x^4}}$ 2 3

$= 2\sec^2 x . \tan x \, e^{\sec^2 x} + \frac{6x}{\sqrt{1-x^4}}$ 1

28 $A = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 0 & 5 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 2 \\ 3 & -4 \\ -5 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} -3-3-10 & 6+4+0 \\ -4-25 & 8+0+0 \end{bmatrix}$ 1

$(AB)' = \begin{bmatrix} -16 & -29 \\ 10 & 8 \end{bmatrix}$ 1/2

3

$B'A' = \begin{bmatrix} -1 & 3 & -5 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -1 & 0 \\ 2 & 5 \end{bmatrix}$ 1

$= \begin{bmatrix} -16 & -29 \\ 10 & 8 \end{bmatrix}$ 1/2

1-9-24-25

(B) $f(x) = \cos x + \sin x$

$f'(x) = \cos x - \sin x$

for c.v.

$f'(x) = 0$

$\cos x = \sin x$

$\tan x = 1$

$x = \pi/4, 5\pi/4 \in (0, 2\pi)$ 1/2

$f''(x) = -\sin x - \cos x$ 1/2

$f''(\frac{\pi}{4}) = -\sin\frac{\pi}{4} - \cos\frac{\pi}{4} = \frac{-2}{\sqrt{2}} < 0$

$\therefore x = \frac{\pi}{4}$ is the point of maxima 1/2

Maximum value $= \cos\frac{\pi}{4} + \sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$ 1/2 3

$f''(\frac{5\pi}{4}) = -\sin\frac{5\pi}{4} - \cos\frac{5\pi}{4}$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} > 0$ 1/2

$\therefore x = \frac{5\pi}{4}$ is the point of L. Minima

Minimum value $= \cos\frac{5\pi}{4} + \sin\frac{5\pi}{4}$

$= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2}$ 1/2

30

31(B) $\int \frac{x^2+x+1}{(x^2+1)(x+2)}\,dx$

$$\frac{x^2+x+1}{(x^2+1)(x+2)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$ 1/2

$$x^2+x+1 = A(x^2+1) + (Bx+C)(x+2)$$

put $x = -2$

$$4-2+1 = 5A \Rightarrow \frac{3}{5} = A$$

$$1 = A + B \qquad 1 = C + 2B$$

$$1 = \frac{3}{5} + B \qquad 1 = C + \frac{4}{5}$$ 1

$$B = 2/5 \qquad \frac{1}{5} = C$$

$$\frac{3}{5}\int \frac{1}{x+2}\,dx + \int \frac{\frac{2}{5}x + \frac{1}{5}}{x^2+1}\,dx$$ 1/2

$$\frac{3}{5}\log|x+2| + \frac{1}{5}\log|x^2+1| + \frac{1}{5}\tan^{-1}x + C$$ 1

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31 $$\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$$

$x = \tan\theta \rightarrow dx = \sec^2\theta\, d\theta$ 1/2

$$\int \cos^{-1}(\cos 2\theta)\sec^2\theta\, d\theta$$ 1

$$2\int \theta \sec^2\theta\, d\theta$$

$$2\left[\theta\tan\theta - \int \tan\theta\, d\theta\right] + C$$ 1

$$2\theta\tan\theta + 2\log|\cos\theta| + C$$

	$2\left[x\tan^{-1}x + \log\frac{1}{\sqrt{1+x^2}}\right]+C$		
	$2\left[x\tan^{-1}x - \log\sqrt{1+x^2}\right]+C$	1/2	3
32 -34(B)	$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$		
	$A^2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$		
	$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$	2	
	$A^2 - 5A + 4I$		5
	$\begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$	2	
	$= \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$	1	

33

33(B)

$x = a(\cos t + \log \tan t/2)$, $y = a\sin t$

$\frac{dx}{dt} = a\left[-\sin t + \frac{\sec^2 t/2}{\tan t/2} \times \frac{1}{2}\right]$ $\frac{dy}{dt} = a\cos t$

$\frac{dx}{dt} = a\left(-\sin t + \frac{1}{\cos^2 t/2} \times \frac{\cos t/2}{\sin t/2} \times \frac{1}{2}\right)$

$= a\left(-\sin t + \frac{1}{\sin t}\right)$

$\frac{d^2y}{dt^2} = -a\sin t$

$= a\left(\frac{1-\sin^2 t}{\sin t}\right)$

$= a\left(\frac{\cos^2 t}{\sin t}\right)$ $1\frac{1}{2}+1$

5

$\frac{\frac{dx}{dt}}{\frac{dy}{dt}} = \frac{\frac{\cancel{a}\cos^2 t}{\sin t}}{\cancel{a}\cos t} = \cot t$ 1

$\frac{dy}{dx} = \tan t$

$\frac{d^2y}{dx^2} = \sec^2 t \frac{dt}{dx}$ 1

$= \sec^2 t \times \frac{\sin t}{a\cos^2 t} = \frac{\sin t}{a\cos^4 t}$ 1/2

$= \frac{1}{a}\sin t \sec^4 t$

or

12024-25

$$x^p y^q = (x+y)^{p+q}$$

Taking log both sides

$$p\log x + q\log y = (p+q)\log(x+y)$$ 1

$$\frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = (p+q)\left[\frac{1}{x+y}\right]\left[1+\frac{dy}{dx}\right]$$ 1

$$\frac{p}{x} - \frac{p+q}{x+y} = \left[\frac{p+q}{x+y} - \frac{q}{y}\right]\frac{dy}{dx}$$

$$\frac{\cancel{px}+py-\cancel{px}-qx}{x\cancel{(x+y)}} = \left[\frac{py+\cancel{qy}-qx-\cancel{qy}}{\cancel{(x+y)}\,y}\right]\frac{dy}{dx}$$ 1

5

$$\frac{\cancel{py-qx}}{x} = \frac{\cancel{py-qx}}{y}\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{y}{x}$$ 1

$$\frac{d^2y}{dx^2} = \frac{x\frac{dy}{dx} - y\times 1}{x}$$

$$= \frac{\cancel{x}\times\frac{y}{\cancel{x}} - y}{x} = 0$$ 1

34 $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

R.h.s

put $a-x=t$

$-dx = dt$

$dx = -dt$

$$-\int_a^0 f(t)\,dt = \int_0^a f(t)\,dt$$

$$= \int_0^a f(x)\,dx$$ $1\frac{1}{2}$

$$I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$$

$x \to \pi - x$

$$I = \int_0^{\pi} \frac{(\pi-x)\sin x}{1+\cos^2 x}\,dx$$ $1/2$

$$2I = \int_0^{\pi} \frac{\cancel{x\sin x} + \pi\sin x - \cancel{x\sin x}}{1+\cos^2 x}\,dx$$ $1/2$

$$2I = \pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$$

put $\cos x = t$ $1/2$

$-\sin x\,dx = dt$

$$2I = -\pi \int_{1}^{-1} \frac{dt}{1+t^2}$$ 1/2

$$2I = -\pi \left[\tan^{-1} t\right]_{+1}^{-1}$$

$$2I = -\pi \left[\tan^{-1}(-1) - \tan^{-1} 1\right]$$ 1 | 5

$$= -\pi \left[-\frac{\pi}{4} - \frac{\pi}{4}\right] = -\pi\left[-\frac{2\pi}{4_2}\right]$$

$$= \frac{\pi^2}{2}$$ 1/2

$$I = \frac{\pi^2}{4}$$

35
35(B)

$A = \{x \in Z : 0 \leq x \leq 12\}$

$R = \{(a, b) : |a-b| \text{ is multiple of } 4\}$

$(a, a) \in R \quad \forall a \in A$

$\because |a-a| = 0$ is multiple of 4 | 1

$\therefore R$ is Reflexive

if $(a, b) \in R$

$\therefore |a-b|$ is multiple of 4

$|-(b-a)|$ " " " 4

$|b-a|$ " " " 4 | $1\frac{1}{2}$

$\therefore (b, a) \in R \quad \forall a, b \in A$

$\therefore R$ is Symmetric

If $(a,b),(b,c) \in R$

$\therefore |a-b|$ is a multiple of 4

$|b-a|$ " " " 4

$|a-b| = 4m \Rightarrow a-b = \pm 4m$

$|b-c| = 4n \Rightarrow b-c = \pm 4n$

$m, n \in Z$

$a - \cancel{b} + \cancel{b} - c = \pm 4(m+n)$

$|a-c| = 4Z \quad [m+n=Z]$ $1\frac{1}{2}$ 5

$\therefore |a-c|$ is multiple of 4

$\therefore R$ is transitive

$\therefore R$ is an equivalence relation

$\{3\} = \{3, 7, 11\}$ 1

36
38(B)

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 41 \\ 29 \\ 44 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$|A| = 3(2-4) - 2(4-4) + 1(4-2)$$

$$= -6 - 0 + 2 = -4 \neq 0$$

1.

(b) (i) $V = x^2 y$ | 1 | 1

(ii) $A = x^2 + 4xy$

$A = x^2 + \frac{4V}{x}$ $\left[y = \frac{V}{x^2}\right]$ | 1 | 1

$\frac{dA}{dx} = 2x - \frac{4V}{x^2}$

$\frac{dA}{dx} = 0$

$\cancel{2}x = \frac{\overset{2}{\cancel{4}}V}{x^2}$

$x^3 = 2V$

$x = (2V)^{1/3}$

$\frac{d^2A}{dx^2} = 2 + \frac{8V}{x^3}$ is +ve when $x^3 = 2V$ | 2 | 2

$\therefore x = (2V)^{1/3}$ is the point of minima.

or

$x^2 y = 1024$ $\quad y = \frac{1024}{x^2}$

$C = 5x^2 + 10xy$

$C = 5x^2 + 10\cancel{x} \times \frac{1024}{x^2}$

2

$\frac{dC}{dx} = 10x + 10240\left(-\frac{1}{x^2}\right)$

$$\text{adj}A = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 2 & -1 \\ 3 & -4 & -1 \end{bmatrix}'$$

$$= \begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix}$$

$$X = -\frac{1}{2}\begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 41 \\ 29 \\ 22 \end{bmatrix}$$

$$X = -\frac{1}{2}\begin{bmatrix} -4 \\ -30 \\ -10 \end{bmatrix} = \begin{bmatrix} 2 \\ 15 \\ 5 \end{bmatrix}$$

(i) Cost of 1 pen = ₹2 — 1

(ii) Cost of 1 pen and 1 sharpener = 2+5 = ₹7 — $1\frac{1}{2}$

(iii) Cost of one pencil & one Sharpener = 15+5 = ₹20 — $1\frac{1}{2}$

or

Cost of one pencil and one Sharpener and one pen = 2+15+5 = ₹22 — $1\frac{1}{2}$

4

	$10x - \frac{10240}{x^2} = 0$ $1\cancel{0}x = \frac{1024\cancel{0}}{x^2}$ $x^3 = 1024$ $x = 8(2)^{1/3}$	2	4
	$\frac{d^2C}{dx^2} = 10 + \frac{10240 \times 2}{x^3}$ is +ve When $x = 8(2)^{1/3}$ ∴ x is the point of minima		
38 37(B)	If $(L_1, L_2) \in R$ (i) ∴ $L_1 \parallel L_2$ $\quad \forall L_1, L_2 \in L$ ∴ $L_2 \parallel L_1$ ∴ $(L_2, L_1) \in R$ ∴ R is Symmetric	$1\frac{1}{2}$	$1\frac{1}{2}$
2	(ii) If $(L_1, L_2), (L_2, L_3) \in R$ ∴ $L_1 \parallel L_2$ & $L_2 \parallel L_3$ ∴ $L_1 \parallel L_3$ $\quad \forall L_1, L_2, L_3 \in L$ ∴ $(L_1, L_3) \in R$ ∴ R is Transitive	$1\frac{1}{2}$	

(iii) $y = 3x + 2$ — (1)

$y = 3x + c$ is $\parallel$ to line (1) for all c — 1

or

if $(L_1, L_2) \in R$

$\therefore L_1 \perp L_2$

$\therefore L_2 \perp L_1 \quad \forall L_1, L_2 \in L$

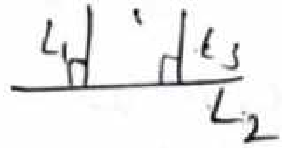
$\therefore (L_2, L_1) \in R$

$\therefore R$ is symmetric — 2

if $(L_1, L_2), (L_2, L_3) \in R$

$\therefore L_1 \perp L_2$ & $L_2 \perp L_3$

but $L_1 \not\perp L_3$



$\because L_1 \parallel L_3 \quad \forall L_1, L_2, L_3 \in L$

$\therefore (L_1, L_3) \notin R$ — 2

$\therefore R$ is not transitive

4

Different questions of Set B

	(b) 8	1	1
	(c) $-2e^{-3x}$	1	1
1.	(d) $-\frac{1}{9}$	1	1
10	(a) 20	1	1
11	(a) $\left[\frac{1}{3}, 1\right]$	1	1
14	(a) $f'(x) < 0 \ \forall \ x \in (a, b)$	1	1
	$\int \frac{dx}{\sqrt{5-4x-x^2}}$ $5-4x-x^2+4-4$ $9-[x^2+4+4x]$ $\int \frac{1}{\sqrt{(3)^2-(x+2)^2}}\, dx$ $9-(x+2)^2$ $(3)^2-(x+2)^2$ $\sin^{-1}\left(\frac{x+2}{3}\right)+C$	1 1	2
23	$\int \frac{(\sin 4x - 4)\, e^x}{1-\cos 4x}\, dx$ $\int \left[\frac{\cancel{2}\sin 2x \cos 2x}{\cancel{2}\sin^2 2x} - \frac{\cancel{4}\,2}{\cancel{2}\sin^2 2x}\right] e^x\, dx$ $\int [\cot 2x - 2\operatorname{cosec}^2 2x]\, e^x\, dx$ $e^x \cot 2x + C$	1/2 1/2 1	2

25 $f(x) = 15 + 12x - 9x^2 + 2x^3$

$f'(x) = 12 - 18x + 6x^2$

$= 6(x^2 - 3x + 2)$

$= 6(x-1)(x-2)$

for c.v.

$f'(x) = 0$ 1/2

$x = 1, 2$

when $x \in (-\infty, 1)$

$f'(x)$ is +ve $\therefore f(x)$ is $\uparrow$ in $(-\infty, 1)$ 1/2

when $x \in (1, 2)$ 2

$f'(x)$ is −ve $\therefore f(x)$ is $\downarrow$ in $(1, 2)$ 1/2

when $x \in (2, \infty)$ $\therefore f'(x)$ is +ve $\therefore$

$f(x)$ is $\uparrow$ in $(2, \infty)$ 1/2

$\boxed{f(x) \text{ is } \downarrow \text{ in } (1,2)}$

26 $AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$

$(AB)' = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$ $1\frac{1}{2}$

$$B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$$ $1\frac{1}{2}$ | 3

$$= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

29

$$\int \tan^{-1}\left(\frac{2x}{1-x^2}\right) dx$$

$x = \tan\theta$ $1/2$

$dx = \sec^2\theta \, d\theta$

$$\int \tan^{-1}(\tan 2\theta) \sec^2\theta \, d\theta$$ $1/2$

$$2\int \theta \sec^2\theta \, d\theta$$

$$2\left[\theta \tan\theta - \int \tan\theta \, d\theta\right] + C$$ $1/2$ | 3

$$2\left[\theta \tan\theta + \log|\cos\theta|\right] + C$$ $1/2$

$$2\left[x \tan^{-1}x + \log\left|\frac{1}{\sqrt{1+x^2}}\right|\right] + C$$ $1/2$

$$2\left[x \tan^{-1}x - \log\sqrt{1-x^2}\right] + C$$ $1/2$

32

$$I = \int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$$

$$x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$$

$$I = \int_0^{\pi/4} \frac{\log(1+\tan\theta)}{\cancel{\sec^2\theta}}\,\cancel{\sec^2\theta}\, d\theta \qquad 1$$

$$\theta \to \pi/4 - \theta$$

$$I = \int_0^{\pi/4} \log\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right) d\theta$$

$$I = \int_0^{\pi/4} \log\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right) d\theta \qquad 1 \qquad 3\tfrac{1}{2}$$

$$I = \int_0^{\pi/4} \log\left(\frac{1+\cancel{\tan\theta}+1-\cancel{\tan\theta}}{1+\tan\theta}\right) d\theta$$

$$I = \int_0^{\pi/4} \log 2 - \log(1+\tan\theta)\, d\theta$$

$$2I = \int_0^{\pi/4} \log 2\, d\theta \qquad 2I = [\theta \log 2]_0^{\pi/4} \qquad 1\tfrac{1}{2}$$

$$2I = \frac{\pi}{4}\log 2$$

$$I = \pi/8 \log 2$$