



General Instructions:-

- 1 All questions are compulsory.
- 2 This question paper has 5 Sections. Section A has 20 questions of 1 mark each which includes 18 M.C.Q.'s and 2 Assertion Reasons Section B has 5 Questions of 2 marks each. Section C has 6 questions of 3 marks each. Section D has 4 questions of 5 mark each and Section E has 3 case study based question of 4 marks each.

Section – A

Q1 If  $A = \{2, 4, 6, 8, 10, 12\}$  then the number of non-empty subsets of set A are

- a)  $2^6$                       b) 6                      c)  $2^6 - 1$                       d)  $2^5$

Q2  $\cos 38^\circ \sin 8^\circ - \sin 38^\circ \cos 8^\circ$  is equal to

- a)  $\frac{1}{2}$                       b)  $-\frac{1}{2}$                       c)  $\frac{\sqrt{3}}{2}$                       d)  $-\frac{\sqrt{3}}{2}$

Q3  $C_9^{11} - C_8^{10} = C_r^{10}$  then r is equal to

- a) 7                      b) 8                      c) 9                      d) 6

Q4  $5.i^{-597}$  in standard form is

- a)  $-5i$                       b)  $0 - 5i$                       c)  $5i$                       d)  $0 + 5i$

Q5 The centre and radius of circle  $3x^2 + 3y^2 = 7$  is

- a)  $(0, 0); \sqrt{7}$                       b)  $(0, 0); \sqrt{3}$                       c)  $(0, 0); \sqrt{\frac{7}{3}}$                       d)  $(0, 0); \frac{7}{3}$

Q6  $\lim_{x \rightarrow \pi} \frac{\tan x}{x - \pi}$

- a)  $-\pi$                       b)  $\pi$                       c) 1                      d) -1

Q7 The minimum value of  $4^{2(1-x)} + 16^x$  is

- a) 8                      b)  $\frac{2}{5}$                       c) 4                      d) 16

Q8  $\lim_{x \rightarrow -a} \frac{x^7 + a^7}{x + a} = 7$ , then the value of a is

- a) 1                      b) -1                      c)  $\pm 1$                       d) 0

Q9 If the extremities of the diagonal of the base of the cube are  $(1, -2, 3)$  and  $(2, -3, 5)$  then the length of the side of the cube is

- a)  $\sqrt{6}$  units                      b)  $\sqrt{3}$  units                      c)  $\sqrt{5}$  units                      d)  $\sqrt{7}$  units

Q10 If  $\tan A = \frac{1}{2}$  and  $\tan B = \frac{1}{3}$  then the value of  $A + B$  is

- a)  $\frac{\pi}{6}$                       b)  $\pi$                       c) 0                      d)  $\frac{\pi}{4}$

Q11 The solution of the inequality :  $5x - 3 < 7$  , when  $x$  is a natural number is

- a) {1}                      b) {1, 2}                      c) (1,2)                      d) {2}

Q12 A relation R in the set of natural numbers is defined as  $R = \{ (x, y): 5x + y = 12 \}$  then the range of the relation R is

- a) {1, 2, 3, 4, 5}                      b) {1, 2}                      c) {7, 2}                      d) {3, 7}

Q13 If point  $(k - 1, 2k, k + 4)$  lies in  $yz$ - plane then its coordinates are

- a)  $(-1, 0, 4)$                       b)  $(0, 2, 4)$                       c)  $(1, 2, 4)$                       d)  $(0, 2, 5)$

Q14 The number of terms in the expansion of  $(a^2 - 2ab + b^2)^{10}$  are

- a) 10                      b) 11                      c) 20                      d) 21

Q15 If E and F are two events associated with a random experiment, having sample space S and

$P(E \cup F) = P(E) + P(F)$ , then which of the following statements is always true

- a)  $E \cup F = S$                       b)  $P(E) = P(F)$                       c)  $P(E \cup F) = 1$                       d)  $E \cap F = \emptyset$

Q16 If  $f(x) = x^2 \sin x$ , then the value of  $\frac{f'(x)}{x}$  is

- a)  $x \cos x + 2 \sin x$                       b)  $x^2 \cos x + 2x \sin x$                       c)  $x \sin x + \cos x$                       d)  $2x \sin x$

Q17 The probability of happening of an event is 0.5 and that of B is 0.3, if A and B are mutually exclusive events then the probability of neither A nor B is

- a) 0.8                      b) 0.2                      c) 0.5                      d) 0.7

Q18 The equation of parabola whose axis is along  $y$  axis, vertex at origin and passing through  $(-2, 5)$  is

- a)  $x^2 = -5y$                       b)  $5y^2 = 4x$                       c)  $5x^2 = 4y$                       d)  $y^2 = -5x$

### Assertion Reason Based Questions:

Choose according to these options in Q 19 and 20

- a) Both A and R are true and R is the correct explanation of A.  
b) Both A and R are true and R is not the correct explanation of A.  
c) A is true and R is false.  
d) A is false and R is true.

Q19 Assertion ( A ) :  $\sin x = \cos x$  for all values of  $x$ .

Reason ( R ) : Trigonometrical Identity is true for all the angles.

Q20 Assertion ( A ) : Distance of point  $(1, 0, -4)$  from  $y$ - axis is  $\sqrt{17}$  units.

Reason ( R ) : Distance of point  $(a, b, c)$  from  $y$ - axis is  $\sqrt{a^2 + c^2}$

### Section – B

Q21 If  $z_1 = 2 - i$ ,  $z_2 = -2 + i$  find  $\text{Im}\left(\frac{z_1 z_2}{z_1}\right)$

Q22 Write the relation  $R = \{(x, x^3) : x \text{ is prime number less than } 10\}$  in roster form.

Q23 Find  $\lim_{x \rightarrow 0} f(x)$ , where  $f(x) = \begin{cases} 2x + 3, & x \leq 0 \\ 3(x + 1), & x > 0 \end{cases}$

OR

Evaluate  $\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1}$

Q24 Find the coordinates of foci, vertices, the eccentricity and the length of latus rectum of the hyperbola

$$9y^2 - 4x^2 = 36$$

OR

Find the equation of circle with centre (2, 2) and passes through the point (4, 5).

Q25 Verify that (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of a right angled triangle.

### Section – C

Q26 A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee has atmost 3 girls?

Q27 Let  $f = \{(x, \frac{x^2}{1+x^2}) : x \in \mathbb{R}\}$  be a function from  $\mathbb{R}$  into  $\mathbb{R}$ . Determine the range of  $R$ .

Q28 Find the value of  $\tan \frac{\pi}{8}$

OR

If  $\cos x = \frac{-1}{3}$ ,  $x$  lies in 3<sup>rd</sup> quadrant then find the value of  $\cos \frac{x}{2}$

Q29 Solve the system of inequalities and represent the solution on number line

$$3x - 7 < 5 + x \quad \text{and} \quad 11 - 5x \leq 1$$

Q30 Find the derivative of  $\frac{x}{\sin^n x}$  with respect to  $x$

Q31 An arch is in the form of a semi-ellipse. It is 8 m wide and 2 m high at the centre. Find the height of the arch at a point 1.5 m from one end.

OR

Find the equation of ellipse with centre (0, 0), major axis on y- axis and passes through the point (3, 2) and (1, 6)

### Section – D

Q32 Find the derivative of  $\frac{\cos x}{x}$  with respect to  $x$  using Ist Principle.

OR

Find the derivative of  $\frac{4ax+5 \sin x}{3bx+7 \cos x}$  with respect to  $x$ .

Q33 If the image of the point (4, 3) with respect to the line  $l_1$  is (2, 1), then find the equation of the line  $l_1$ . Also find the value of k if the distance between the above line and the line  $3x + 3y + k = 0$  is  $\frac{14}{\sqrt{3}}$  units.

OR

Find the image of the point (3, 8) with respect to the line  $x + 3y = 7$  assuming the line to be a plane mirror.

Q34 Given  $A = \{x: x \in \mathbb{R}, \text{ and } x \text{ satisfy } x^2 - 8x + 12 = 0\}$

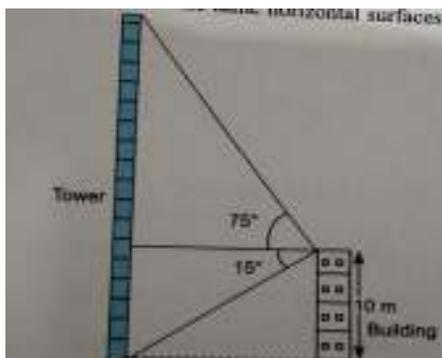
$B = \{x : x \in \mathbb{R}, \text{ and } 2 \leq x \leq 7\}$

Find i)  $A \cap B$                       ii)  $A - B$                       iii)  $A' \cap B$

Q35 Find the coefficient of  $x^5$  in  $(1 + 2x)^6(1 - x)^7$  using binomial theorem.

### Section – E

Q36 From the top of a 10 m high building the angle of elevation of top of a tower is  $75^\circ$  and the angle of depression of foot of tower is  $15^\circ$ . If the tower and building are on the same horizontal surfaces.



- i) Find the value of  $\tan 15^\circ$ .
- ii) Find the distance between the foot of the tower and the foot of the building.
- iii) Find the value of  $\cos 75^\circ$ .

OR

Find the height of the tower.

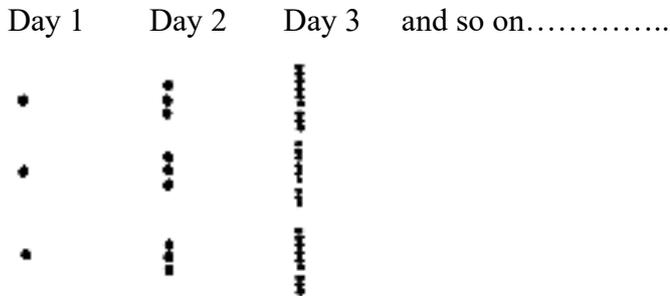
Q37 Many candidates apply for a job in a company. Company short listed few candidates. The particulars of candidates are as follows:

S. No.	Name	Sex	Age (in years)
1	Sheetal	F	30
2	Ramesh	M	33
3	Meena	F	46
4	Alis	M	28
5	Akbar	M	41

If two persons are selected at random. What is the probability that

- i) Both are male
- ii) Both are female
- iii) One is male and one is female

Q38 On the first day of new year i.e. on 1 January Ramesh helped 3 persons. When those persons thanked him, he advised them not to thank but to help 3 more persons on second day and instruct them to do the same on third day. They move the chain similarly.



Assuming the chain is not broken, answer the following:

- i) Find how many persons will be helped on 5<sup>th</sup> day.
- ii) Find the total number of people helped in 5 days.
- iii) 6,561 persons will be helped on which day.





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- 1 All questions are compulsory.
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Section – A

Q1 The centre and radius of circle  $2x^2 + 2y^2 - x = 0$  is

- a)  $(\frac{1}{4}, 0); 1$       b)  $(\frac{-1}{4}, 0); \sqrt{3}$       c)  $(0, 0); \sqrt{\frac{1}{4}}$       d)  $(\frac{1}{4}, 0); \frac{1}{4}$

Q2 If E and F are two events associated with a random experiment, having sample space S and  $P(E \cup F) = P(E) + P(F)$ , then which of the following statements is always true

- a)  $E \cup F = S$       b)  $P(E) = P(F)$       c)  $P(E \cup F) = 1$       d)  $E \cap F = \emptyset$

Q3 The equation of hyperbola with foci  $(0, \pm 4)$  and length of transverse axis as 6 is

- a)  $\frac{x^2}{9} - \frac{y^2}{7} = 1$       c)  $\frac{x^2}{7} - \frac{y^2}{9} = 1$   
b)  $\frac{y^2}{9} - \frac{x^2}{7} = 1$       d)  $\frac{y^2}{7} - \frac{x^2}{9} = 1$

Q4 The number of terms in the expansion of  $(x^2 + 6x + 9)^{12}$  are

- a) 11      b) 12      c) 25      d) 24

Q5 If  $\tan A = \frac{1}{2}$  and  $\tan B = \frac{1}{3}$  then the value of  $A + B$  is

- a)  $\frac{\pi}{6}$       b)  $\pi$       c) 0      d)  $\frac{\pi}{4}$

Q6  $\lim_{x \rightarrow 1} \frac{x^{15} - 1}{x^{10} - 1}$  is

- a) 1      b)  $\frac{3}{2}$       c) 2      d) 0

Q7  $\cos 57^\circ \sin 3^\circ + \sin 57^\circ \cos 3^\circ$  is equal to

- a)  $\frac{1}{2}$       b)  $\frac{-1}{2}$       c)  $\frac{\sqrt{3}}{2}$       d)  $\frac{-\sqrt{3}}{2}$

Q8 The minimum value of  $3^{2(1-x)} + 9^x$  is

- a) 9      b)  $\frac{2}{5}$       c) 4      d) 6

Q9 If  $A = \{2, 4, 6, 8, 10, 12\}$  then the number of subsets of set A are

- a)  $2^6$                       b) 6                      c)  $2^6 - 1$                       d)  $2^5$

Q10 The solution of the inequality :  $-8 \leq 5x - 3 < 7$  where  $x \in \mathbb{R}$  is

- a)  $[-1, 2)$                       b)  $\{-1, 2\}$                       c)  $(-1, 2)$                       d)  $[2, \infty)$

Q11  $C_6^{13} - C_5^{12} = C_r^{12}$  then r is equal to

- a) 7                      b) 8                      c) 9                      d) 6

Q12 If the extremities of the diagonal of the base of the cube are  $(-1, 2, -3)$  and  $(-2, 3, 5)$  then the length of the side of the cube is

- a)  $\sqrt{66}$  units                      b)  $\sqrt{33}$  units                      c)  $\sqrt{50}$  units                      d)  $\sqrt{70}$  units

Q13  $(1 - i)^{-2}$  in standard form is

- a)  $-2i$                       b)  $0 + \frac{i}{2}$                       c)  $0 - \frac{i}{2}$                       d)  $0 - 2i$

Q14  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$

- a) -2                      b) 2                      c) 1                      d) -1

Q15 If point  $(k - 1, 2k, k + 4)$  lies in  $xz$ - plane then its coordinates are

- a)  $(-1, 0, 4)$                       b)  $(0, 2, 4)$                       c)  $(1, 2, 4)$                       d)  $(0, 2, 5)$

Q16 If  $P = \{1, 2, 3, 4, \dots, 14\}$ . A relation R from P to P is defined by

$R = \{(x, y) : 3x - y = 0, \text{ where } x, y \in P\}$ . The domain of relation R is

- a)  $\{1, 2, 3\}$                       b) P                      c)  $\{3, 6, 9, 12\}$                       d)  $\{1, 2, 3, 4\}$

Q17 If  $f(x) = x^3 \cos x$ , then the value of  $\frac{f'(x)}{x}$  is

- a)  $x^3 \sin x + 3x^2 \cos x$                       b)  $-x^3 \sin x + 3x^2 \cos x$                       c)  $-x^2 \sin x + 3x \cos x$                       d)  $x \sin x$

Q18 The probability of happening of an event is 0.5 and that of B is 0.3, if A and B are mutually exclusive events then the probability of neither A nor B is

- a) 0.8                      b) 0.2                      c) 0.5                      d) 0.7

**Assertion Reason Based Questions: Choose according to these options in Q 19 and 20**

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Q19 Assertion (A) : Distance of point  $(1, 0, -4)$  from  $y$ - axis is  $\sqrt{17}$  units.

Reason (R) : Distance of point  $(a, b, c)$  from  $y$ - axis is  $\sqrt{a^2 + c^2}$

Q20 Assertion (A) :  $\sin x = \cos x$  for all values of  $x$ .

Reason (R) : Trigonometrical Identity is true for all the angles.

### Section - B

Q21 Let  $f(x) = 2x^2 + 3x - 5$  and  $g(x) = x - 1$ . Find  $\left(\frac{f}{g}\right)(x)$ . Also find the domain and range of quotient function.

Q22 Verify that  $(0, 7, -10)$ ,  $(1, 6, -6)$  and  $(4, 9, -6)$  are the vertices of an isosceles triangle.

Q23 Find  $\lim_{x \rightarrow 1} f(x)$ , where  $f(x) = \begin{cases} x^2 - 1, & x \leq 1 \\ -x^2 - 1, & x > 1 \end{cases}$

OR

Evaluate  $\lim_{x \rightarrow 0} \frac{\sin ax + bx}{ax + \sin bx}$

Q24 Find the coordinates of foci, vertices, the eccentricity and the length of latus rectum of the ellipse

$$36x^2 + 4y^2 = 144$$

OR

Find the equation of circle with radius 5 whose centre lies on  $x$  - axis and passes through the point  $(2, 3)$

Q25 If  $z_1 = 2 - i$ ,  $z_2 = 1 + i$  find  $\operatorname{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$

### Section - C

Q26 Find the derivative of  $\frac{x^2 \cos \frac{\pi}{4}}{\sin x}$  with respect to  $x$

Q27 Let  $f = \left\{ \left(x, \frac{x^2}{1+x^2}\right) : x \in \mathbb{R} \right\}$  be a function from  $\mathbb{R}$  into  $\mathbb{R}$ . Determine the range of  $\mathbb{R}$ .

Q28 Find the area of triangle formed by the lines joining the vertex of the parabola  $x^2 = 12y$  to the ends of its latus rectum.

OR

Find the equation of hyperbola with foci  $(\pm 4, 0)$  and the length of latus rectum 12.

Q29 A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee has at least 3 girls?

Q30 Solve the system of inequalities and represent the solution on number line

$$37 - (3x + 5) \geq 9x - 8(x - 3)$$

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OR

If  $\cos x = \frac{-1}{3}$ ,  $x$  lies in 3<sup>rd</sup> quadrant then find the value of  $\sin \frac{x}{2}$

### Section - D

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OR

Find the derivative of  $\frac{3ax+5 \cos x}{4bx+7 \sin x}$  with respect to x.

Q34 Find the image of the point (3, 8) with respect to the line  $x + 3y = 7$  assuming the line to be a plane mirror.

OR

If the image of the point (4, 3) with respect to the line  $l_1$  is (2, 1), then find the equation of the line  $l_1$ . Also find the value of k if the distance between the above line and the line  $3x + 3y + k = 0$  is  $\frac{14}{\sqrt{3}}$  units.

Q35 Given  $U = \{1, 2, 3, 4, 5, \dots, 20\}$ ,  $A = \{x: x \in Z, x^2 - 3x + 2 = 0\}$ ,

$B = \{1, 3, 5, 7\}$  find i)  $A - B$  ii)  $A \cap B$  iii)  $A' \cap B$

### Section – E

Q36 Many candidates apply for a job in a company. Company short listed few candidates. The particulars of candidates are as follows:

S. No.	Name	Sex	Age (in years)
1	Sheetal	F	30
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If two persons are selected at random. What is the probability that

- i) Both are male
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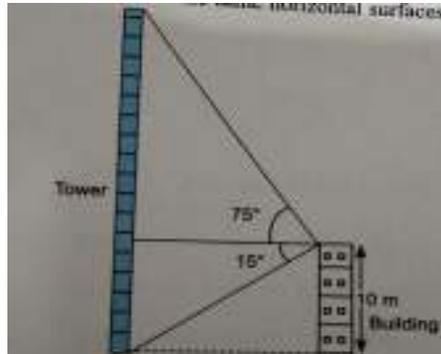
Day 1      Day 2      Day 3      and so on.....



Assuming the chain is not broken, answer the following:

- i) Find how many persons will be helped on 5<sup>th</sup> day.
- ii) Find the total number of people helped in 5 days.
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- i) Find the value of  $\tan 15^\circ$ .
- ii) Find the distance between the foot of the tower and the foot of the building.
- iii) Find the value of  $\cos 75^\circ$ .

OR

Find the height of the tower.



Note:- Any relevant solution not given here but done by the students will be suitably awarded

Q-no	Value Points / Key Points	Value Point	Total Point
1	c) $2^4 - 1$		
2	b) $-\frac{1}{2}$		
3	c) 9		
4	b) $0 - 5i$		
5	c) $(1, 0), \sqrt{3}$		
6	c) 1		
7	a) 8		
8	c) $\pm 1$		
9	b) $\sqrt{3}$		
10	d) $\pi/4$		
11	a) $\{1\}$		
12	c) $\{7, 2\}$		
13	d) $(0, 2, 5)$		
14	d) 21		
15	d) $E \cap F = \emptyset$		
16	a) $x \cos x + 2 \sin x$		
17	b) 0.2		
18	c) $5x^2 = 4y$		
19	d) A is false and R is true		
20	a) Both A and R are true and R is the correct explanation of A		

Section-B

Q21  $\frac{z_1 z_2}{z_1} = \frac{(2-i)(-2+i)}{2-i} = \frac{-4+2i+2i-i^2}{2-i} = \frac{-3+4i}{2-i}$   
 $= \frac{-3+4i}{2-i} \times \frac{2+i}{2+i} = \frac{-6+3i+8i-4i^2}{4-i^2} = \frac{-2+11i}{5}$   
 $\text{Im}\left(\frac{z_1 z_2}{z_1}\right) = \frac{11}{5}$

Q22  $R = \{(x, x^3) : x \text{ is prime no. less than } 10\}$   
 $= \{(2, 8), (3, 27), (5, 125), (7, 343)\}$

Q23  $f(x) = \begin{cases} 2x+3, & x \leq 0 \\ 3(x+1), & x > 0 \end{cases}$

LHL  $\lim_{x \rightarrow 0^-} 2x+3 = \lim_{h \rightarrow 0} 2(0-h)+3 = 3$

RHL  $\lim_{x \rightarrow 0^+} 3(x+1) = \lim_{h \rightarrow 0} 3(0+h+1) = 3$   
 $\Rightarrow \lim_{x \rightarrow 0} f(x) = 3$

$\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1} = \lim_{x \rightarrow 0} \frac{-2\sin^2 x}{-2\sin^2 \frac{x}{2}}$   
 $= \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sin^2 \frac{x}{2}} \times \frac{x^2}{x^2} = \left(\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2}\right) \times \left(\lim_{x \rightarrow 0} \frac{x^2 \times 4}{3x^2}\right)$   
 $= 1 \times 4 = 4$

Q24  $9y^2 - 4x^2 = 36$

$\frac{y^2}{4} - \frac{x^2}{9} = 1$  Compare with  $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$a^2 = 4, b^2 = 9$

Foci =  $(0, \pm c) = (0, \pm \sqrt{13})$

Vertices =  $(0, \pm a) = (0, \pm 2)$

Length of Latus Rectum =  $\frac{2b^2}{a} = \frac{2 \times 9}{2} = 9$

Eccentricity =  $e = \frac{c}{a} = \frac{\sqrt{13}}{2}$

OR  
 (21)  $(4, 5)$   
 $R = \sqrt{(4-2)^2 + (5-2)^2} = \sqrt{4+9} = \sqrt{13}$   
 $(x-2)^2 + (y-2)^2 = (\sqrt{13})^2$   
 $x^2 + 4 - 4x + y^2 + 4 - 4y = 13$   
 $x^2 + y^2 - 4x - 4y - 5 = 0$

025  
 $A(0, 7, 10), B(-1, 6, 6), C(-4, 9, 6)$

$$AB = \sqrt{(-1)^2 + (6-7)^2 + (6-10)^2} = \sqrt{1+1+16} = \sqrt{18} \quad \frac{1}{2}$$

$$BC = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2} = \sqrt{9+9+0} = \sqrt{18} \quad \frac{1}{2}$$

$$AC = \sqrt{(-4)^2 + (7-7)^2 + (6-10)^2} = \sqrt{16+4+16} = \sqrt{36} = 6 \quad \frac{1}{2}$$

Now  $(AC)^2 = 36$

$$(AB)^2 + (BC)^2 = (\sqrt{18})^2 + (\sqrt{18})^2 = 18 + 18 = 36$$

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$

$\Rightarrow A, B, C$  are vertices of right angle  $\Delta$

Section-C

026  
 $9B, 4C$

At most 3 girls  $\Rightarrow$  0G 7B, 1G 6B, 2G 5B, 3G 4B

$$\begin{aligned} \text{No. of ways} &= {}^9C_0 \times {}^4C_0 + {}^9C_1 \times {}^4C_1 + {}^9C_2 \times {}^4C_2 + {}^9C_3 \times {}^4C_3 \\ &= \frac{9!}{7!2!} \times \frac{4!}{3!1!} + \frac{9!}{8!1!} \times \frac{4!}{3!1!} + \frac{9!}{7!2!} \times \frac{4!}{2!2!} + \frac{9!}{6!3!} \times \frac{4!}{4!0!} \end{aligned} \quad \frac{1}{2}$$

$$= \frac{9 \times 8}{2} + \frac{9 \times 8 \times 7 \times 4}{2 \times 2} + \frac{9 \times 8 \times 7 \times 6}{2 \times 2} + \frac{9 \times 8 \times 7 \times 6}{3 \times 2} \quad \frac{1}{2} \quad 3$$

$$= 36 + 336 + 756 + 504$$

$$= 1632$$

027  
 $f(x) = \frac{x^2}{1+x^2}, D_f = \mathbb{R}$

$$y = \frac{x^2}{1+x^2} \Rightarrow y(1+x^2) = x^2$$

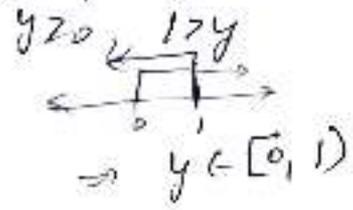
$$\Rightarrow x^2(y-1) = -y$$

$$\Rightarrow x^2 = \frac{-y}{y-1} = \frac{y}{1-y}$$

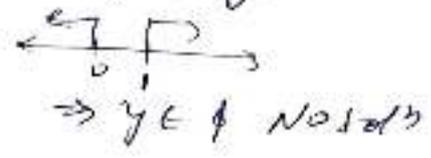
$$\Rightarrow x^2 = \frac{y}{1-y}$$

As  $x^2 \geq 0$  always  $\Rightarrow \frac{y}{1-y} \geq 0$

Case I  
 $y \geq 0, 1-y > 0$



Case II  
 $y \leq 0, 1-y < 0$   
 $y \leq 0, 1 < y$



$$\Rightarrow y \in [0, 1)$$

Range of  $f(x) = [0, 1)$

$\frac{1}{2}$

1+1

$\frac{1}{2}$

2

3

3

To find  $\tan \frac{\pi}{8}$

We know  $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

Put  $x = \frac{\pi}{8}$

$\tan 2 \times \frac{\pi}{8} = \frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}}$

$\tan \frac{\pi}{4} = \frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}}$

$1 = \frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}}$  Put  $\tan \frac{\pi}{8} = y$

$1 = \frac{2y}{1-y^2} \Rightarrow 1-y^2 = 2y$   
 $\Rightarrow y^2 + 2y - 1 = 0$   
 $\Rightarrow y = \frac{-2 \pm \sqrt{4+4}}{2}$

$\Rightarrow y = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$

$\Rightarrow \tan \frac{\pi}{8} = -1 + \sqrt{2}$  or  $(-1 - \sqrt{2})$  rejected

$\Rightarrow \boxed{\tan \frac{\pi}{8} = -1 + \sqrt{2}}$

because  $\tan \frac{\pi}{8}$  is not -ve in I<sup>st</sup> quadrant

$\cos x = -\frac{1}{3}$ ,  $x$  lies in III<sup>rd</sup> quadrant

$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2} = \frac{1 - \frac{1}{3}}{2} = \frac{\frac{2}{3}}{2} = \frac{1}{3}$

so,  $\cos \frac{x}{2} = \pm \frac{1}{\sqrt{3}}$

$\frac{180 \leq x \leq 270}{2} \Rightarrow 90 \leq \frac{x}{2} \leq 135$

$\Rightarrow \frac{x}{2}$  lies in II<sup>nd</sup> quadrant

$\cos \theta$  is -ve in II<sup>nd</sup> quadrant

$\Rightarrow \boxed{\cos \frac{x}{2} = -\frac{1}{\sqrt{3}}}$

Q29

$3n-7 < 5+x$  and  $11-5n \leq 1$

$3n-x < 5+7$  and  $-5n \leq 1-11$

$2n < 12$  and  $-5n \leq -10$

$n < 6$  and  $5n \geq 10$

$\Rightarrow 2 \leq n < 6$   $n \geq 2$

$\Rightarrow x \in [2, 6)$

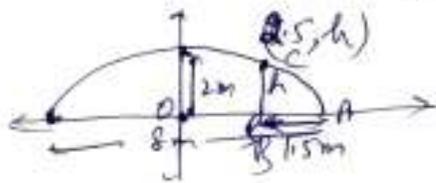
030

$$y = \frac{x}{\sin^n x}$$

$$\frac{dy}{dx} = \frac{\sin^n x \times \frac{d}{dx}(x) - x \times \frac{d}{dx}(\sin^n x)}{(\sin^n x)^2}$$

$$= \frac{\sin^n x - x \times n \sin^{n-1} x \times \frac{d}{dx}(\sin x)}{\sin^{2n} x}$$

$$\frac{dy}{dx} = \frac{\sin^n x - nx \sin^{n-1} x \cos x}{\sin^{2n} x}$$



Let eqn of ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Here  $a=4$   $\Rightarrow$   $\frac{x^2}{16} + \frac{y^2}{4} = 1$  (1)

Now let height of arc that is 1.5m from one end =  $h = AB$

$$\Rightarrow OB = 4 - 1.5 = 2.5 \text{ m}$$

Coordinates of point C are  $(2.5, h)$

$(2.5, h)$  point lies of (1)

So,  $\frac{(2.5)^2}{16} + \frac{h^2}{4} = 1$

$$\frac{h^2}{4} = 1 - \frac{(2.5)^2}{16} = \frac{16 - 6.25}{16}$$

$$= 1 - \frac{6.25}{16}$$

$$\frac{h^2}{4} = \frac{16 - 6.25}{16}$$

$$h^2 = \frac{9.75}{4} \times 4$$

$$h^2 = 2.43$$

$$h = \sqrt{2.43} =$$

$$= 1.5$$

So, height = 1.5 m

$$\begin{array}{r} 1.5 \\ 1 \overline{) 2.43} \\ \underline{1} \phantom{0} \\ 1 \phantom{0} \\ \underline{1} \phantom{0} \\ 0 \phantom{0} \end{array}$$

1/2

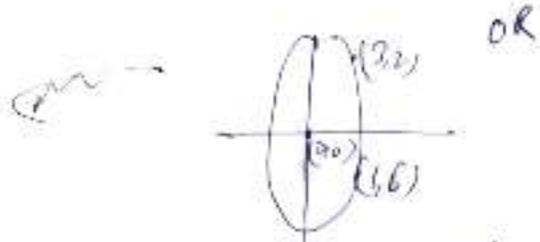
1/2 3

1/2

1/2

1

1 3



Let eq<sup>n</sup> of ellipse is

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1 \quad \text{where } a > b$$

$$\Rightarrow \frac{(0)^2}{a^2} + \frac{(3)^2}{b^2} = 1$$

$$\Rightarrow \frac{9}{b^2} + \frac{0}{a^2} = 1$$

$$\frac{9}{a^2} + \frac{1}{b^2} = 1$$

Put  $\frac{1}{a^2} = u$ ,  $\frac{1}{b^2} = v$

$$9(4u + 9v = 1)$$

$$36u + 81v = 9$$

$$36u + v = 1$$

$$36u + 81v = 9$$


---


$$-80v = -8$$

$$\boxed{v = \frac{1}{10}}$$

$$u = \frac{1 - 9v}{4}$$

$$= \frac{1 - \frac{9}{10}}{4} = \frac{1}{40}$$

$$\boxed{u = \frac{1}{40}}$$

$$\Rightarrow a^2 = \frac{1}{u} = 40$$

and  $b^2 = \frac{1}{v} = 10$

$\Rightarrow$  Eq<sup>n</sup> of ellipse is  $\boxed{\frac{y^2}{40} + \frac{x^2}{10} = 1}$

Section-D.

033

$$f(x) = \frac{\cos x}{x}$$

so,  $f(x+h) = \frac{\cos(x+h)}{x+h}$

now  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \frac{\frac{\cos(x+h)}{x+h} - \frac{\cos x}{x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x \cos(x+h) - (x+h) \cos x}{x(x+h)h}$$

$$= \lim_{h \rightarrow 0} \frac{x(\cos x \cos h - \sin x \sin h) - x \cos x - h \cos x}{x(x+h)h}$$

$$= \lim_{h \rightarrow 0} \frac{x \cos x (\cos h - 1) - x \sin x \sin h - h \cos x}{x(x+h)h}$$

1/2

1/2

1

3

$$= \lim_{h \rightarrow 0} \frac{x \cos x (\cosh h - 1)}{x(x+h)h} - \lim_{h \rightarrow 0} \frac{x \sin x \sinh h}{x(x+h)h}$$

$$= \lim_{h \rightarrow 0} \frac{x \cos x}{x(x+h)h}$$

We know  $\left( \lim_{h \rightarrow 0} \frac{\cosh h - 1}{h} \right) = 0$ ,  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$= 0 \times 1 - \frac{\sin x}{x} - \frac{\cos x}{x(x+0)}$$

$$= -\frac{\sin x}{x} - \frac{\cos x}{x^2}$$

$$\Rightarrow \boxed{f'(x) = -\frac{\sin x}{x} - \frac{\cos x}{x^2}}$$

OR

$$y = \frac{4ax + 5 \sin x}{3bx + 7 \cos x}$$

$$\frac{dy}{dx} = \frac{(3bx + 7 \cos x) \frac{d}{dx}(4ax + 5 \sin x) - (4ax + 5 \sin x) \frac{d}{dx}(3bx + 7 \cos x)}{(3bx + 7 \cos x)^2}$$

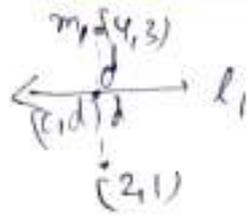
$$= \frac{(3bx + 7 \cos x)(4a + 5 \cos x) - (4ax + 5 \sin x)(3b - 7 \sin x)}{(3bx + 7 \cos x)^2}$$

$$= \frac{12abx + 15b \cos x + 28a \cos x + 35 \cos^2 x - [12abx - 28ax \sin x + 15b \sin x - 35 \sin^2 x]}{(3bx + 7 \cos x)^2}$$

$$= \frac{35(\cos^2 x + \sin^2 x) + (28a \cos x + 28ax \sin x) + 15b \cos x - 15b \sin x}{(3bx + 7 \cos x)^2}$$

$$= \frac{35 + 28a \cos x + 28ax \sin x - 15b \sin x + 15b \cos x}{(3bx + 7 \cos x)^2}$$

23



Let eqn of line \$l\_1\$ is \$ax+by+c=0\$

Slope of line \$m\_1\$ is  $\frac{1-3}{2-4} = \frac{-2}{-2} = 1$

Now line \$l\_1\$ and \$m\_1\$ are \$\perp\$  
So, Slope of line \$l\_1 \times\$ Slope of line \$m\_1 = -1\$

\$\Rightarrow -\frac{a}{b} \times 1 = -1\$

\$\Rightarrow -a = -b\$

\$\Rightarrow \boxed{a=b}\$

Eqn of line \$m\_1\$ is \$y-3 = 1(x-4)\$

\$y-3 = x-4\$

\$y-x-3+4=0\$

\$\Rightarrow \boxed{y-x+1=0}\$

Let \$(c, d)\$ is the point of intersection of \$l\_1\$ and \$m\_1\$,  
Now \$(c, d)\$ is mid point of line \$m\_1\$,

So, \$c = \frac{4+2}{2}\$, \$d = \frac{3+1}{2}\$

\$c = \frac{6}{2}\$, \$d = \frac{4}{2}\$

\$c = 3\$, \$d = 2\$

\$\rightarrow (3, 2)\$ is passing point of line \$l\$  
and slope is  $-\frac{a}{b} = -\frac{a}{a} = -1$$

So, Eqn of line \$l\_1\$ is

\$y-2 = -1(x-3)\$

\$y-2 = -x+3\$

\$\boxed{y+x-5=0}\$

\$\boxed{y+x-5=0}\$

\$d = \frac{14}{\sqrt{2}}\$

\$3x+2y+k=0\$

\$\boxed{2x+y+k=0}\$

\$d = \frac{|c\_1 - c\_2|}{\sqrt{a^2 + b^2}}\$

\$\frac{14}{\sqrt{2}} = \frac{|5 - k|}{\sqrt{1+1}}\$

\$\frac{14}{\sqrt{2}} = \frac{5+k}{\sqrt{2}} = \frac{15+k}{\sqrt{2}}\$

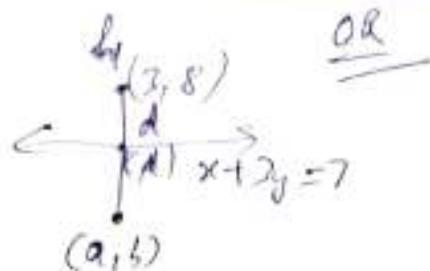
\$42\sqrt{2} = 15\sqrt{2} + \sqrt{2}k \Rightarrow \boxed{k = \frac{42\sqrt{2} - 15\sqrt{2}}{\sqrt{2}}}\$

2

1

1

1 5



Let  $(c, d)$  is point of intersection

Now  $(c, d)$  is mid point of line  $l_1$

$$\rightarrow (c, d) = \left( \frac{3+a}{2}, \frac{8+b}{2} \right)$$

Slope of line  $x+2y=7$  is  $-\frac{1}{2}$

Slope of line  $l_1$  is  $\frac{b-8}{a-3}$

Now  $m_1 m_2 = -1$

$$\Rightarrow \frac{1}{2} \left( \frac{b-8}{a-3} \right) = -1$$

$$\Rightarrow b-8 = 3a-9$$

$$\Rightarrow b-3a-8+9=0$$

$$\Rightarrow b-3a+1=0$$

$$\Rightarrow \boxed{3a-b-1=0}$$

$$\begin{aligned} \text{distance } d \text{ is} &= \frac{|3+3 \times 8-7|}{\sqrt{1+9}} \\ &= \frac{|3+24-7|}{\sqrt{10}} \\ &= \frac{20}{\sqrt{10}} \end{aligned}$$

$$d = \frac{|a+3b-7|}{\sqrt{1+9}} = \frac{|a+3b-7|}{\sqrt{10}}$$

$$\Rightarrow \frac{a+3b-7}{\sqrt{10}} = \frac{20}{\sqrt{10}}$$

$$\Rightarrow a+3b=27$$

$$3(a-b)=1$$

$$9a-3b=3$$

$$\frac{10a=30}{a=3}$$

$$b=8$$

$$a+3b-7=-20$$

$$a+3b=-20+7$$

$$3(a+b)=-13$$

$$3a-b=1$$

$$3a+9b=-39$$

$$\frac{-10b=40}{b=-4}$$

$$3a=1-4$$

$$a=-1$$

$$\Rightarrow \text{Mirror Image} = (-1, -4)$$

Q34

$$A = \{x: x \in R, x^2 - 8x + 12 = 0\}$$

$$= \{x: x \in R, x^2 - 6x - 2x + 12 = 0\}$$

$$= \{x: x \in R, (x-6)(x-2) = 0\}$$

$$= \{2, 6\}$$

$$B = \{x: x \in R, 2 \leq x \leq 7\}$$

$$= [2, 7]$$

- (i)  $A \cap B = \{2, 6\}$
- (ii)  $A - B = \phi$
- (iii)  $A' \cap B = B - A = (2, 6) \cup (6, 7]$

$\frac{1}{2}$   
 $\frac{1}{2}$  5  
 2

Q35

$$(1+2x)^6 (1-x)^7$$

$$(1+x)^n = nC_0 + nC_1 x + nC_2 x^2 + nC_3 x^3 + \dots + nC_n x^n$$

$$\text{So, } (1+2x)^6 = 6C_0 + 6C_1 (2x) + 6C_2 (2x)^2 + \dots + 6C_6 (2x)^6$$

$$(1-x)^7 = 7C_0 + 7C_1 (-x) + 7C_2 (-x)^2 + \dots + 7C_7 (-x)^7$$

$$\text{Now } (1+2x)^6 (1-x)^7 = \{6C_0 + 6C_1 (2x) + 6C_2 (2x)^2 + \dots + 6C_6 (2x)^6\}$$

$$\{7C_0 + 7C_1 (-x) + 7C_2 (-x)^2 + \dots + 7C_7 (-x)^7\}$$

To find Coefficient of  $x^5$  are  $[6C_5 (2x)^5 7C_0 + 6C_4 (2x)^4 7C_1 (-x) +$   
 $6C_3 (2x)^3 7C_2 (-x)^2 + 6C_2 (2x)^2 7C_3 (-x)^3 +$   
 $6C_1 (2x) 7C_4 (-x)^4 + 6C_0 (2x)^0 7C_5 (-x)^5]$

$$\text{Coefficients of } x^5 = (2)^5 6C_5 7C_0 - 6C_4 (2)^4 7C_1 + 6C_3 (2)^3 7C_2 -$$

$$6C_2 (2)^2 7C_3 + 6C_1 (2) 7C_4 - 6C_0 7C_5$$

$$= 32 \times \frac{6!}{5!1!} \times \frac{7!}{0!7!} - \frac{6!}{4!2!} (2)^4 \times \frac{7!}{1!6!} +$$

$$\frac{6!}{3!3!} \times 8 \times \frac{7!}{2!5!} - \frac{6!}{2!4!} \times 4 \times \frac{7!}{3!4!} + \frac{6!}{1!5!} \times 2 \times \frac{7!}{4!3!} - \frac{6!}{0!7!} \times \frac{7!}{5!2!}$$

$$= 32 \times 6 - 6 \times 5 \times 8 \times 7 + \frac{6 \times 5 \times 4 \times 8 \times 7 \times 6 \times 5}{3 \times 2} - \frac{6 \times 5 \times 4 \times 7 \times 6 \times 5}{2} + 6 \times 2 \times \frac{7 \times 6 \times 5}{3 \times 2} - \frac{6 \times 7}{1 \times 2}$$

$$= 32 \times 6 - 6 \times 5 \times 56 + 20 \times 8 \times 21 - 10 \times 7 \times 30 + 14 \times 30 - 21$$

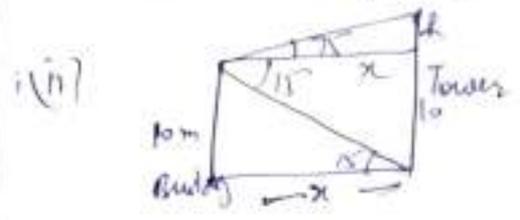
$$= 192 - 1680 + 3360 - 2100 + 420 - 21$$

$$= 171$$

5

Section - E

Q36 (i)  $\tan 15 = \tan(45-30) = \frac{1-\tan 30}{1+\tan 30} = \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$



$\frac{10}{x} = \tan 15 = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$x = \frac{10(\sqrt{3}+1)}{\sqrt{3}-1}$

(ii)  $\cos 75 = \cos(45+30)$   
 $= \cos 45 \cos 30 - \sin 45 \sin 30$   
 $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$   
 $= \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\frac{h}{x} = \tan 75 = \tan(90-15) = \cot 15 = \frac{1}{\tan 15}$

$\frac{h}{x} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$h = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{10(\sqrt{3}+1)}{\sqrt{3}-1} = \frac{10(3+1+2\sqrt{3})}{3-1}$

$= \frac{10(4+2\sqrt{3})}{2} = 5(4+2\sqrt{3}) = 20+10\sqrt{3}$

Total candidates = 5  
 No. of males = 3  
 No. of females = 2

(i) Probability that both are males =  $\frac{{}^3C_2}{{}^5C_2}$   
 $= \frac{\frac{3!}{2!1!}}{\frac{5!}{2!3!}} = \frac{3!}{2!} \times \frac{2!3!}{5!}$   
 $= \frac{3 \times 2 \times 2 \times 2}{5 \times 4 \times 3 \times 2}$   
 $= \frac{3}{10}$

(ii) Probability that both are female =  $\frac{{}^2C_2}{{}^5C_2}$   
 $= \frac{1}{\frac{5!}{2!3!}} = \frac{2!3!}{5!}$   
 $= \frac{1}{10}$

(iii) Probability that one is male and one is female =  $\frac{{}^3C_1 \times {}^2C_1}{{}^5C_2} = \frac{3 \times 2}{\frac{5!}{2!3!}} = \frac{3 \times 2 \times 2 \times 2}{5 \times 4 \times 3 \times 2}$   
 $= \frac{3}{5}$

(38)

$3, 3^2, 3^3, \dots$

(i) It is GP with  $a=3, r=3$

So,  $a_5 = ar^4 = 3 \times (3)^4 = 3^5$

(ii)  $S_n = \frac{a(r^n - 1)}{r - 1}$

$S_5 = \frac{a(r^5 - 1)}{r - 1} = \frac{3(3^5 - 1)}{3 - 1} = \frac{3}{2}(3^5 - 1)$

(iii)  $a_n = 6561$

$a_n = ar^{n-1}$

$6561 = 3 \times (3)^{n-1} = 3^n$

$(3)^8 = 3^n$

$n = 8$  Ans.

So, 6561 persons will be helped on 8<sup>th</sup> day.

3	6561
3	2187
9	729
27	27
81	9

2

4

Note:- Any relevant solution not given herein but done by the students is suitably awarded.

Q.No.	Value Points / Key Points	Value Point	Total Point
1	d) $(\frac{1}{4}, 0); \frac{1}{4}$	1	
2	d) $\emptyset \cap A = \emptyset$	1	
3	b) $\frac{y^2}{9} - \frac{x^2}{7} = 1$	1	
4	c) 25	1	
5	d) $\pi/4$	1	
6	b) $3/2$	1	
7	c) $\sqrt{3}/2$	1	
8	d) 6	1	
9	a) 26	1	
10	a) $E(1, 2)$	1	
11	d) 6	1	
12	a) $\sqrt{66}$	1	
13	b) $0 + \frac{1}{2}$	1	
14	b) 2	1	
15	a) $(-1, 0, 4)$	1	
16	d) $\{1, 2, 3, 4\}$	1	
17	c) $-x^2 \sin x + 3x \cos x$	1	
18	b) 0.2	1	
19	a) Both A and R are true and R is the correct explanation of A	1	
20	d) A is false and R is true.	1	

021

Section-B

$$f(x) = 2x^2 + 3x - 5, \quad g(x) = x - 1$$

$$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{2x^2 + 3x - 5}{x - 1} = \frac{2x^2 + 5x - 2x - 5}{x - 1}$$

$$= \frac{x(2x + 5) - 1(2x + 5)}{x - 1}$$

$$= \frac{(2x + 5)(x - 1)}{x - 1} = 2x + 5$$

$$\Rightarrow \frac{f}{g}(x) = 2x + 5$$

$$\text{Domain of } \frac{f}{g}(x) = \text{Domain of } \frac{2x^2 + 3x - 5}{x - 1} = \mathbb{R} - \{1\}$$

$$\text{Range of } \frac{f}{g}(x) = \dots$$

$$y = 2x + 5$$

$$2x = y - 5$$

$$x = \frac{y - 5}{2}$$

$$\text{Here } x \neq 1 \Rightarrow \frac{y - 5}{2} \neq 1$$

$$y \neq 7$$

$$\Rightarrow \text{Range of } \frac{f}{g}(x) = \mathbb{R} - \{7\}$$

022

$$AB = \sqrt{(1-0)^2 + (1-7)^2 + (-6+1)^2}$$

$$= \sqrt{1+1+16} = \sqrt{18}$$

$$BC = \sqrt{(4-1)^2 + (9-0)^2 + (-6+6)^2}$$

$$= \sqrt{9+9} = \sqrt{18}$$

$$AC = \sqrt{(4-0)^2 + (9-7)^2 + (-6+1)^2}$$

$$= \sqrt{16+4+16} = \sqrt{36} = 6$$

Here  $AB = BC$ , so, A, B, C are vertices of isosceles  $\Delta$ .

022

$$\text{L.H.L. } \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x^2 - 1 = \lim_{h \rightarrow 0} (1-h)^2 - 1 = 1 - 1 = 0$$

$$\text{R.H.L. } \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} -x^2 - 1 = \lim_{h \rightarrow 0} -(1+h)^2 - 1 = -2$$

So, L.H.L.  $\neq$  R.H.L.  
So, Limit does not exist

$$\lim_{x \rightarrow 0} \frac{\sin ax + bx}{ax + \sin bx} = \lim_{x \rightarrow 0} \left( \frac{\sin ax}{ax} \times ax \right) + \lim_{x \rightarrow 0} bx$$

$$= \lim_{x \rightarrow 0} (ax + bx) = \lim_{x \rightarrow 0} (a + b)x = 0$$

24

$$\frac{x^2}{4} + \frac{y^2}{36} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a^2 = 36, b^2 = 4$$

$$c^2 = a^2 - b^2 = 36 - 4 = 32$$

$$c = \pm 4\sqrt{2}$$

$$\text{foci} = (0, \pm c) = (0, \pm 4\sqrt{2})$$

$$\text{vertices} = (0, \pm a) = (0, \pm 6)$$

$$e = \frac{c}{a} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$$

$$\text{length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times 4}{6} = \frac{4}{3}$$

$\frac{1}{2}$  each 2

25



OR

$$(a-2)^2 + (0-0)^2 = 25$$

$$(a-2)^2 = 25 - 0 = 25$$

$$a^2 + 4 - 4a = 25$$

$$a^2 - 4a - 21 = 0$$

$$a^2 - 6a + 2a - 21 = 0$$

$$(a-6)(a+2) = 0$$

$$a = -2, 6$$

Case I

$$\text{Centre} = (-2, 0)$$

Eq<sup>n</sup> of Circle  $\rightarrow$

$$(x+2)^2 + y^2 = 25$$

$$x^2 + y^2 + 4x - 21 = 0$$

1

$\frac{1}{2}$

Case II Centre (6, 0)

Eq<sup>n</sup>

$$(x-6)^2 + y^2 = 25$$

$$x^2 + y^2 - 12x + 11 = 0$$

$\frac{1}{2}$

2

26

$$z_1 z_2 = (2-i)(1+i) = 2 + 2i - i - 1 = 1 + i$$

$$\frac{z_1 z_2}{z_1} = \frac{2+i}{2-i} \times \frac{2+i}{2+i} = \frac{6 - 3i + 2i - 1}{4 - i^2} = \frac{5 - i}{5}$$

$$= \frac{5-i}{5}$$

$$\text{Re}\left(\frac{z_1 z_2}{z_1}\right) = \frac{5}{5} = 1$$

$\frac{1}{2}$

$\frac{1}{2}$

2

Section-C

$$y = \frac{x^2 \cos x}{\sin x} = \frac{x^2}{\sin x} \cos x$$

$$\frac{dy}{dx} = \frac{1}{\sin^2 x} \left[ \sin x \times 2x - x^2 \cos x \right]$$

$$= \frac{2x \sin x - x^2 \cos x}{\sin^2 x}$$

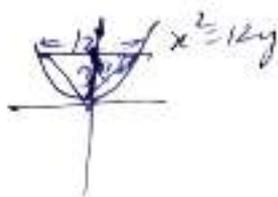
$\frac{1}{2}$

2

$\frac{1}{2}$

3

27



$$x^2 = 4ay$$

$$4a = 12$$

$$a = 3 \quad \text{focus} = (0, a) = (0, 3)$$

$$\text{Area} = \frac{1}{2} \times b \times h = \frac{1}{2} \times 12 \times 3 = 18$$

or

$$\text{foci} = (\pm 4, 0) = (\pm c, 0) \quad [c = 4]$$

$$\frac{2b^2}{a} = 12$$

$$[2b^2 = 12a]$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$c^2 = a^2 + b^2$$

$$16 = a^2 + b^2$$

When  $a = -8$

$$b^2 = 60 = 6 \times 10$$

$$= -48$$

rejected

$$a^2 + b^2 - 16 = 0$$

$$a^2 + 8a - 2a - 16 = 0$$

$$(a+8)(a-2) = 0$$

$$a = -8, 2$$

Also when  $a = 2$

$$b^2 = 60 = 12$$

$$\Rightarrow$$

$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$

(79)

9B, 4G (No. of) ways when committee has at least 3 girls

$$= {}^3C_6 4B + 4G 3B$$

$$= {}^4C_2 \times {}^9C_4 + 4C_4 \times {}^9C_3$$

$$= 504 + 84$$

$$= 588$$

(80)

$$37 - (3x + 5) \geq 9x - 8(x - 2)$$

$$37 - 3x - 5 \geq 9x - 8x + 16$$

$$32 - 24 \geq x + 16$$

$$8 \geq 4x$$

$$x \leq 2 \Rightarrow x \in (-\infty, 2]$$



2

1

2

1

3

3

3

3

31

OR

$$\cos x = -\frac{1}{3} \quad 180^\circ \leq x \leq 270^\circ$$

$$90^\circ \leq \frac{x}{2} \leq 135^\circ$$

$$\therefore \sin \frac{x}{2} = \frac{1 - \cos x}{2} \Rightarrow \frac{x}{2} \text{ lies in II<sup>nd</sup> quadrant}$$

$$= \frac{1 + \frac{1}{3}}{2} = \frac{\frac{4}{3} \times \frac{1}{2}}{1} = \frac{2}{3}$$

$$\sin \frac{x}{2} = \pm \sqrt{\frac{2}{3}}$$

$$\Rightarrow \sin \frac{x}{2} = \sqrt{\frac{2}{3}} \text{ as } \frac{x}{2} \text{ lies in II<sup>nd</sup> quadrant}$$

OR

$$y = \frac{3ax + 5 \cos x}{4bx + 7 \sin x}$$

$$\frac{dy}{dx} = \frac{(4bx + 7 \sin x)(3a - 5 \sin x) - (3ax + 5 \cos x)(4b + 7 \cos x)}{(4bx + 7 \sin x)^2}$$

$$= \frac{(4bx + 7 \sin x)(3a - 5 \sin x) - (3ax + 5 \cos x)(4b + 7 \cos x)}{(4bx + 7 \sin x)^2}$$

$$= \frac{(12bxa - 20bx \sin x + 21a \sin x - 35 \sin^2 x) - (120bx + 21ax \cos x + 20b \cos x + 35 \cos^2 x)}{(4bx + 7 \sin x)^2}$$

$$= \frac{-20bx \sin x + 21a \sin x - 35 \sin^2 x - 21ax \cos x - 20b \cos x - 35 \cos^2 x}{(4bx + 7 \sin x)^2}$$

$$= \frac{21a \sin x - 20bx \sin x - 21ax \cos x - 35 - 20b \cos x}{(4bx + 7 \sin x)^2}$$

$$= \frac{21a(\sin x - x \cos x) - 20b(x \sin x + \cos x) - 35}{(4bx + 7 \sin x)^2}$$