



Time: 3 Hrs .

M.M. : 80

General Instructions:-

- 1 All questions are compulsory.
- 2 This question paper has 5 Sections. Section A has 20 questions of 1 mark each which includes 18 M.C.Q.'s and 2 Assertion Reasons Section B has 5 Questions of 2 marks each. Section C has 6 questions of 3 marks each. Section D has 4 questions of 5 mark each and Section E has 3 case study based question of 4 marks each.

Section – A

Q1 If $A = \{2, 4, 6, 8, 10, 12\}$ then the number of non-empty subsets of set A are

- a) 2^6 b) 6 c) $2^6 - 1$ d) 2^5

Q2 $\cos 38^\circ \sin 8^\circ - \sin 38^\circ \cos 8^\circ$ is equal to

- a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) $\frac{\sqrt{3}}{2}$ d) $-\frac{\sqrt{3}}{2}$

Q3 $C_9^{11} - C_8^{10} = C_r^{10}$ then r is equal to

- a) 7 b) 8 c) 9 d) 6

Q4 $5.i^{-597}$ in standard form is

- a) $-5i$ b) $0 - 5i$ c) $5i$ d) $0 + 5i$

Q5 The centre and radius of circle $3x^2 + 3y^2 = 7$ is

- a) $(0, 0); \sqrt{7}$ b) $(0, 0); \sqrt{3}$ c) $(0, 0); \sqrt{\frac{7}{3}}$ d) $(0, 0); \frac{7}{3}$

Q6 $\lim_{x \rightarrow \pi} \frac{\tan x}{x - \pi}$

- a) $-\pi$ b) π c) 1 d) -1

Q7 The minimum value of $4^{2(1-x)} + 16^x$ is

- a) 8 b) $\frac{2}{5}$ c) 4 d) 16

Q8 $\lim_{x \rightarrow -a} \frac{x^7 + a^7}{x + a} = 7$, then the value of a is

- a) 1 b) -1 c) ± 1 d) 0

Q9 If the extremities of the diagonal of the base of the cube are $(1, -2, 3)$ and $(2, -3, 5)$ then the length of the side of the cube is

- a) $\sqrt{6}$ units b) $\sqrt{3}$ units c) $\sqrt{5}$ units d) $\sqrt{7}$ units

Q10 If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$ then the value of $A + B$ is

- a) $\frac{\pi}{6}$ b) π c) 0 d) $\frac{\pi}{4}$

Q11 The solution of the inequality : $5x - 3 < 7$, when x is a natural number is

- a) $\{1\}$ b) $\{1, 2\}$ c) $(1, 2)$ d) $\{2\}$

Q12 A relation R in the set of natural numbers is defined as $R = \{ (x, y): 5x + y = 12 \}$ then the range of the relation R is

- a) $\{1, 2, 3, 4, 5\}$ b) $\{1, 2\}$ c) $\{7, 2\}$ d) $\{3, 7\}$

Q13 If point $(k - 1, 2k, k + 4)$ lies in yz - plane then its coordinates are

- a) $(-1, 0, 4)$ b) $(0, 2, 4)$ c) $(1, 2, 4)$ d) $(0, 2, 5)$

Q14 The number of terms in the expansion of $(a^2 - 2ab + b^2)^{10}$ are

- a) 10 b) 11 c) 20 d) 21

Q15 If E and F are two events associated with a random experiment, having sample space S and

$P(E \cup F) = P(E) + P(F)$, then which of the following statements is always true

- a) $E \cup F = S$ b) $P(E) = P(F)$ c) $P(E \cup F) = 1$ d) $E \cap F = \emptyset$

Q16 If $f(x) = x^2 \sin x$, then the value of $\frac{f'(x)}{x}$ is

- a) $x \cos x + 2 \sin x$ b) $x^2 \cos x + 2x \sin x$ c) $x \sin x + \cos x$ d) $2x \sin x$

Q17 The probability of happening of an event is 0.5 and that of B is 0.3, if A and B are mutually exclusive events then the probability of neither A nor B is

- a) 0.8 b) 0.2 c) 0.5 d) 0.7

Q18 The equation of parabola whose axis is along y axis, vertex at origin and passing through $(-2, 5)$ is

- a) $x^2 = -5y$ b) $5y^2 = 4x$ c) $5x^2 = 4y$ d) $y^2 = -5x$

Assertion Reason Based Questions:

Choose according to these options in Q 19 and 20

- a) Both A and R are true and R is the correct explanation of A .
b) Both A and R are true and R is not the correct explanation of A .
c) A is true and R is false.
d) A is false and R is true.

Q19 Assertion (A) : $\sin x = \cos x$ for all values of x .

Reason (R) : Trigonometrical Identity is true for all the angles.

Q20 Assertion (A) : Distance of point $(1, 0, -4)$ from y - axis is $\sqrt{17}$ units.

Reason (R) : Distance of point (a, b, c) from y - axis is $\sqrt{a^2 + c^2}$

Section – B

Q21 If $z_1 = 2 - i$, $z_2 = -2 + i$ find $\text{Im}\left(\frac{z_1 z_2}{z_1}\right)$

Q22 Write the relation $R = \{(x, x^3) : x \text{ is prime number less than } 10\}$ in roster form.

Q23 Find $\lim_{x \rightarrow 0} f(x)$, where $f(x) = \begin{cases} 2x + 3, & x \leq 0 \\ 3(x + 1), & x > 0 \end{cases}$

OR

Evaluate $\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1}$

Q24 Find the coordinates of foci, vertices, the eccentricity and the length of latus rectum of the hyperbola

$$9y^2 - 4x^2 = 36$$

OR

Find the equation of circle with centre (2, 2) and passes through the point (4, 5).

Q25 Verify that (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of a right angled triangle.

Section – C

Q26 A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee has atmost 3 girls?

Q27 Let $f = \{(x, \frac{x^2}{1+x^2}) : x \in \mathbb{R}\}$ be a function from \mathbb{R} into \mathbb{R} . Determine the range of R .

Q28 Find the value of $\tan \frac{\pi}{8}$

OR

If $\cos x = \frac{-1}{3}$, x lies in 3rd quadrant then find the value of $\cos \frac{x}{2}$

Q29 Solve the system of inequalities and represent the solution on number line

$$3x - 7 < 5 + x \quad \text{and} \quad 11 - 5x \leq 1$$

Q30 Find the derivative of $\frac{x}{\sin^n x}$ with respect to x

Q31 An arch is in the form of a semi-ellipse. It is 8 m wide and 2 m high at the centre. Find the height of the arch at a point 1.5 m from one end.

OR

Find the equation of ellipse with centre (0, 0), major axis on y- axis and passes through the point (3, 2) and (1, 6)

Section – D

Q32 Find the derivative of $\frac{\cos x}{x}$ with respect to x using Ist Principle.

OR

Find the derivative of $\frac{4ax+5 \sin x}{3bx+7 \cos x}$ with respect to x .

Q33 If the image of the point (4, 3) with respect to the line l_1 is (2, 1), then find the equation of the line l_1 . Also find the value of k if the distance between the above line and the line $3x + 3y + k = 0$ is $\frac{14}{\sqrt{3}}$ units.

OR

Find the image of the point (3, 8) with respect to the line $x + 3y = 7$ assuming the line to be a plane mirror.

Q34 Given $A = \{x: x \in \mathbb{R}, \text{ and } x \text{ satisfy } x^2 - 8x + 12 = 0\}$

$$B = \{x : x \in \mathbb{R}, \text{ and } 2 \leq x \leq 7\}$$

Find i) $A \cap B$

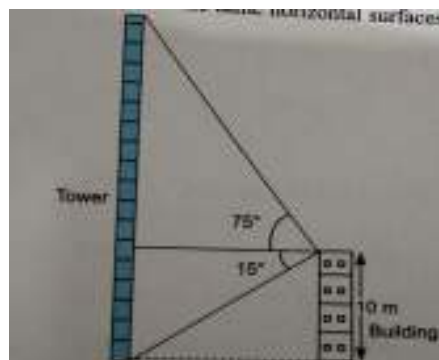
ii) $A - B$

iii) $A' \cap B$

Q35 Find the coefficient of x^5 in $(1 + 2x)^6(1 - x)^7$ using binomial theorem.

Section – E

Q36 From the top of a 10 m high building the angle of elevation of top of a tower is 75° and the angle of depression of foot of tower is 15° . If the tower and building are on the same horizontal surfaces.



- Find the value of $\tan 15^\circ$.
- Find the distance between the foot of the tower and the foot of the building.
- Find the value of $\cos 75^\circ$.

OR

Find the height of the tower.

Q37 Many candidates apply for a job in a company. Company short listed few candidates. The particulars of candidates are as follows:

S. No.	Name	Sex	Age (in years)
1	Sheetal	F	30
2	Ramesh	M	33
3	Meena	F	46
4	Alis	M	28
5	Akbar	M	41

If two persons are selected at random. What is the probability that

- i) Both are male
- ii) Both are female
- iii) One is male and one is female

Q38 On the first day of new year i.e. on 1 January Ramesh helped 3 persons. When those persons thanked him, he advised them not to thank but to help 3 more persons on second day and instruct them to do the same on third day. They move the chain similarly.

Day 1 Day 2 Day 3 and so on.....

The diagram illustrates the experimental setup. A participant is seated at a table, looking at a screen. On the screen, there is a 3x3 grid of dots. A cursor is positioned at the center dot. The participant is instructed to move the cursor to the dots. The diagram includes labels for the participant, the screen, the cursor, and the dots.

Assuming the chain is not broken, answer the following:

- Find how many persons will be helped on 5th day.
- Find the total number of people helped in 5 days.
- 6,561 persons will be helped on which day.



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Section – A

Q1 The centre and radius of circle $2x^2 + 2y^2 - x = 0$ is

- a) $(\frac{1}{4}, 0); 1$ b) $(\frac{-1}{4}, 0); \sqrt{3}$ c) $(0, 0); \sqrt{\frac{1}{4}}$ d) $(\frac{1}{4}, 0); \frac{1}{4}$

Q2 If E and F are two events associated with a random experiment, having sample space S and $P(E \cup F) = P(E) + P(F)$, then which of the following statements is always true

- a) $E \cup F = S$ b) $P(E) = P(F)$ c) $P(E \cup F) = 1$ d) $E \cap F = \emptyset$

Q3 The equation of hyperbola with foci $(0, \pm 4)$ and length of transverse axis as 6 is

- a) $\frac{x^2}{9} - \frac{y^2}{7} = 1$ c) $\frac{x^2}{7} - \frac{y^2}{9} = 1$
b) $\frac{y^2}{9} - \frac{x^2}{7} = 1$ d) $\frac{y^2}{7} - \frac{x^2}{9} = 1$

Q4 The number of terms in the expansion of $(x^2 + 6x + 9)^{12}$ are

- a) 11 b) 12 c) 25 d) 24

Q5 If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$ then the value of $A + B$ is

- a) $\frac{\pi}{6}$ b) π c) 0 d) $\frac{\pi}{4}$

Q6 $\lim_{x \rightarrow 1} \frac{x^{15} - 1}{x^{10} - 1}$ is

- a) 1 b) $\frac{3}{2}$ c) 2 d) 0

Q7 $\cos 57^\circ \sin 3^\circ + \sin 57^\circ \cos 3^\circ$ is equal to

- a) $\frac{1}{2}$ b) $\frac{-1}{2}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{-\sqrt{3}}{2}$

Q8 The minimum value of $3^{2(1-x)} + 9^x$ is

- a) 9 b) $\frac{2}{5}$ c) 4 d) 6

Q9 If $A = \{2, 4, 6, 8, 10, 12\}$ then the number of subsets of set A are

- a) 2^6 b) 6 c) $2^6 - 1$ d) 2^5

Q10 The solution of the inequality : $-8 \leq 5x - 3 < 7$ where $x \in \mathbb{R}$ is

- a) $[-1, 2)$ b) $\{-1, 2\}$ c) $(-1, 2)$ d) $[2, \infty)$

Q11 $C_6^{13} - C_5^{12} = C_r^{12}$ then r is equal to

- a) 7 b) 8 c) 9 d) 6

Q12 If the extremities of the diagonal of the base of the cube are $(-1, 2, -3)$ and $(-2, 3, 5)$ then the length of the side of the cube is

- a) $\sqrt{66}$ units b) $\sqrt{33}$ units c) $\sqrt{50}$ units d) $\sqrt{70}$ units

Q13 $(1 - i)^{-2}$ in standard form is

- a) $-2i$ b) $0 + \frac{i}{2}$ c) $0 - \frac{i}{2}$ d) $0 - 2i$

Q14 $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$

- a) -2 b) 2 c) 1 d) -1

Q15 If point $(k - 1, 2k, k + 4)$ lies in xz- plane then its coordinates are

- a) $(-1, 0, 4)$ b) $(0, 2, 4)$ c) $(1, 2, 4)$ d) $(0, 2, 5)$

Q16 If $P = \{1, 2, 3, 4, \dots, 14\}$. A relation R from P to P is defined by

$R = \{(x, y) : 3x - y = 0, \text{ where } x, y \in P\}$. The domain of relation R is

- a) $\{1, 2, 3\}$ b) P c) $\{3, 6, 9, 12\}$ d) $\{1, 2, 3, 4\}$

Q17 If $f(x) = x^3 \cos x$, then the value of $\frac{f'(x)}{x}$ is

- a) $x^3 \sin x + 3x^2 \cos x$ b) $-x^3 \sin x + 3x^2 \cos x$ c) $-x^2 \sin x + 3x \cos x$ d) $x \sin x$

Q18 The probability of happening of an event is 0.5 and that of B is 0.3, if A and B are mutually exclusive events then the probability of neither A nor B is

- a) 0.8 b) 0.2 c) 0.5 d) 0.7

Assertion Reason Based Questions: Choose according to these options in Q 19 and 20

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Section - B

Q21 Let $f(x) = 2x^2 + 3x - 5$ and $g(x) = x - 1$. Find $\left(\frac{f}{g}\right)(x)$. Also find the domain and range of quotient function.

Q22 Verify that $(0, 7, -10)$, $(1, 6, -6)$ and $(4, 9, -6)$ are the vertices of an isosceles triangle.

Q23 Find $\lim_{x \rightarrow 1} f(x)$, where $f(x) = \begin{cases} x^2 - 1, & x \leq 1 \\ -x^2 - 1, & x > 1 \end{cases}$

OR

Evaluate $\lim_{x \rightarrow 0} \frac{\sin ax + bx}{ax + \sin bx}$

Q24 Find the coordinates of foci, vertices, the eccentricity and the length of latus rectum of the ellipse

$$36x^2 + 4y^2 = 144$$

OR

Find the equation of circle with radius 5 whose centre lies on x – axis and passes through the point $(2, 3)$

Q25 If $z_1 = 2 - i$, $z_2 = 1 + i$ find $\operatorname{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$

Section – C

Q26 Find the derivative of $\frac{x^2 \cos \frac{\pi}{4}}{\sin x}$ with respect to x

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Q28 Find the area of triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum.

OR

Find the equation of hyperbola with foci $(\pm 4, 0)$ and the length of latus rectum 12.

Q29 A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee has atleast 3 girls?

Q30 Solve the system of inequalities and represent the solution on number line

$$37 - (3x + 5) \geq 9x - 8(x - 3)$$

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If $\cos x = \frac{-1}{3}$, x lies in 3rd quadrant then find the value of $\sin \frac{x}{2}$

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Q35 Given $U = \{1, 2, 3, 4, 5, \dots, 20\}$, $A = \{x: x \in Z, x^2 - 3x + 2 = 0\}$,

$B = \{1, 3, 5, 7\}$ find i) $A - B$ ii) $A \cap B$ iii) $A' \cap B$

Section – E

Q36 Many candidates apply for a job in a company. Company short listed few candidates. The particulars of candidates are as follows:

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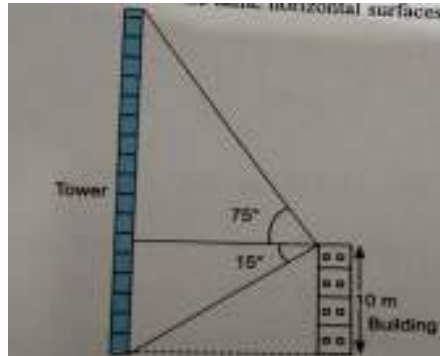
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- i) Find how many persons will be helped on 5th day.
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Q38 From the top of a 10 m high building the angle of elevation of top of a tower is 75° and the angle of depression of foot of tower is 15° . If the tower and building are on the same horizontal surfaces.



- i) Find the value of $\tan 15^\circ$.
- ii) Find the distance between the foot of the tower and the foot of the building.
- iii) Find the value of $\cos 75^\circ$.

OR

Find the height of the tower.

Note:- Any relevant solution not given here but done by the students will be suitably awarded

Q. No.	Value Points / Key Points	Value Point	Total Point
1	c) $2^6 - 1$	1	
2	b) $-\frac{1}{2}$	1	
3	c) 9	1	
4	b) $0 - 5i$	1	
5	c) $(1, 0), \sqrt{3}$	1	
6	c) 1	1	
7	a) 8	1	
8	c) ± 1	1	
9	b) $\sqrt{3}$	1	
10	d) $\pi/4$	1	
11	a) $\{1\}$	1	
12	c) $\{7, 2\}$	1	
13	d) $(0, 2, 5)$	1	
14	d) 21	1	
15	d) $E \cap F = \emptyset$	1	
16	a) $x \cos x + 2 \sin x$	1	
17	b) 0.2	1	
18	c) $5x^2 = 4y$	1	
19	d) A is false and R is true	1	
20	a) Both A and R are true and R is the correct explanation of A	1	

Section-B

Q21

$$\frac{z_1 z_2}{z_1} = \frac{(2-i)(-2+i)}{2-i} = \frac{-4+2i+2i-i^2}{2-i} = \frac{-3+4i}{2-i}$$

$$= \frac{-3+4i}{2-i} \times \frac{2+i}{2+i} = \frac{-6+3i+8i-4i^2}{4-i^2} = \frac{-2+11i}{5}$$

$$\operatorname{Im}\left(\frac{z_1 z_2}{z_1}\right) = \frac{11}{5}$$

Q22

$$R = \{(x, x^3) : x \text{ is prime no. less than } 10\}$$

$$= \{(2, 8), (3, 27), (5, 125), (7, 343)\}$$

Q23

$$f(x) = \begin{cases} 2x+3, & x \leq 0 \\ 3(x+1), & x > 0 \end{cases}$$

$$\text{LHL: } \lim_{x \rightarrow 0^-} 2x+3 = \lim_{x \rightarrow 0^-} 2(0-h)+3 = 3$$

$$\text{RHL: } \lim_{x \rightarrow 0^+} 3(x+1) = \lim_{x \rightarrow 0^+} 3(0+h+1) = 3$$

$$\Rightarrow \lim_{x \rightarrow 0} f(x) = 3$$

$$\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1} = \lim_{x \rightarrow 0} \frac{-2 \sin^2 x}{-2 \sin^2 \frac{x}{2}}$$

$$= \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sin^2 \frac{x}{2}} \times \frac{x^2}{x^2} = \left(\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \right) \times \left(\lim_{x \rightarrow 0} \frac{x^2}{\sin^2 \frac{x}{2}} \right)$$

$$= 1 \times 4 = 4$$

Q24

$$9y^2 - 4x^2 = 36$$

$$\frac{y^2}{4} - \frac{x^2}{9} = 1 \quad \text{Compare with } \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$a^2 = 4, \quad b^2 = 9$$

$$\text{Foci} = (0, \pm c) = (0, \pm \sqrt{13})$$

$$\text{Vertices} = (0, \pm a) = (0, \pm 2)$$

$$\text{Length of Latus Rectum} = \frac{2b^2}{a} = \frac{2 \times 9}{2} = 9$$

$$\text{Eccentricity} = e = \frac{c}{a} = \frac{\sqrt{13}}{2}$$

$$\text{OR}$$

$$\text{Circumference} = \sqrt{(4-2)^2 + (5-2)^2} = \sqrt{4+9} = \sqrt{13}$$

$$(x-2)^2 + (y-5)^2 = (\sqrt{13})^2$$

$$x^2 + 4 - 4x + y^2 + 25 - 10y = 13$$

$$x^2 + y^2 - 4x - 10y + 16 = 0$$

Q25: $A(0, 7, 10), B(-1, 6, 6), C(-4, 9, 6)$

$$AB = \sqrt{(-1)^2 + (6-7)^2 + (6-10)^2} = \sqrt{1+1+16} = \sqrt{18} \quad \frac{1}{2}$$

$$BC = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2} = \sqrt{9+9+0} = \sqrt{18} \quad \frac{1}{2}$$

$$AC = \sqrt{(-4)^2 + (7-7)^2 + (10-6)^2} = \sqrt{16+0+16} = \sqrt{32} = 4\sqrt{2} \quad \frac{1}{2}$$

Now $(AC)^2 = 32$

$$(AB)^2 + (BC)^2 = (\sqrt{18})^2 + (\sqrt{18})^2 = 18+18=36 \quad \frac{1}{2}$$

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$

$\Rightarrow A, B, C$ are vertices of right angle Δ

Section-C

Q26: 9B, 4C

At most 3 girls \Rightarrow 0G 7B, 1G 6B, 2G 5B, 3G 4B

No. of ways = ${}^9C_0 \times {}^4C_0 + {}^9C_1 \times {}^4C_1 + {}^9C_2 \times {}^4C_2 + {}^9C_3 \times {}^4C_3 \quad \frac{1}{2}$

$$= \frac{9!}{7!2!} \times \frac{4!}{0!4!} + \frac{9!}{8!1!} \times \frac{4!}{3!1!} + \frac{9!}{7!2!} \times \frac{4!}{2!2!} + \frac{9!}{6!3!} \times \frac{4!}{4!0!}$$

$$= \frac{9 \times 8}{2} + \frac{9 \times 8 \times 7 \times 4}{2 \times 2} + \frac{9 \times 8 \times 7 \times 6}{2 \times 2} + \frac{9 \times 8 \times 7 \times 6}{3 \times 2} \quad \frac{1}{2} \quad 3$$

$$= 36 + 336 + 756 + 504$$

$$= 1632$$

Q27: $f(x) = \frac{x^2}{1+x^2}, D_f = \mathbb{R}$

$$y = \frac{x^2}{1+x^2} \Rightarrow y(1+x^2) = x^2$$

$$\Rightarrow x^2(y-1) = -y$$

$$\Rightarrow x^2 = \frac{-y}{y-1} = \frac{y}{1-y}$$

$$\Rightarrow x^2 = \frac{y}{1-y}$$

As $x^2 \geq 0$ always $\Rightarrow \frac{y}{1-y} \geq 0$

Case I

$$y \geq 0, 1-y > 0$$

$$y \geq 0, 1 > y$$



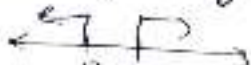
$$\Rightarrow y \in [0, 1)$$

$$\Rightarrow y \in (0, 1)$$

Case II

$$y \leq 0, 1-y < 0$$

$$y \leq 0, 1 < y$$



$$\Rightarrow y \in \emptyset \text{ No soln}$$

Range of $f(x) = (0, 1)$

$$\frac{1}{2} \quad 3$$

030

$$y = \frac{x}{\sin^n x}$$

$$\frac{dy}{dx} = \frac{\sin^n x \times \frac{d}{dx}(x) - x \times \frac{d}{dx}(\sin^n x)}{(\sin^n x)^2}$$

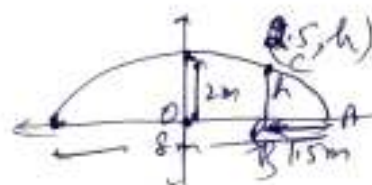
$$= \frac{\sin^n x - x \times n \sin^{n-1} x \times \frac{d}{dx}(\sin x)}{\sin^{2n} x}$$

$$\frac{dy}{dx} = \frac{\sin^n x - nx \sin^{n-1} x \cos x}{\sin^{2n} x}$$

1 1/2

1 1/2 3

031



Let eqn of ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{Here } a=4, b=2 \Rightarrow \frac{x^2}{16} + \frac{y^2}{4} = 1 \quad \text{--- (1)}$$

1 1/2

Now let height of arc that is 1.5m from one end = $h = AB$

$$\Rightarrow OB = 4 - 1.5 = 2.5 \text{ m}$$

Coordinates of point C are $(2.5, h)$

$(2.5, h)$ point lies on (1)

1 1/2

$$\text{So, } \frac{(2.5)^2}{16} + \frac{h^2}{4} = 1$$

$$\frac{h^2}{4} = 1 - \frac{(2.5)^2}{16} = \frac{16 - 6.25}{16}$$

1

$$= 1 - \frac{6.25}{16}$$

$$\frac{h^2}{4} = \frac{16 - 6.25}{16}$$

$$h^2 = \frac{9.75}{4}$$

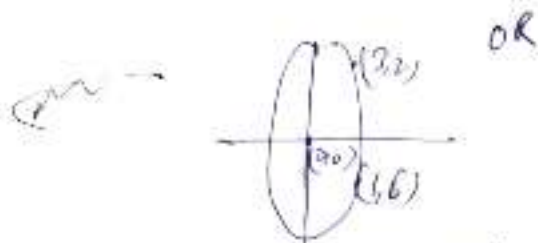
$$h^2 = 2.43$$

$$h = \sqrt{2.43} = 1.5$$

So, height = 1.5 m

$$\begin{array}{r} 1.5 \\ 1 \overline{) 2.43} \\ \underline{1} \\ 1 43 \\ \underline{1} 43 \\ 0 \end{array}$$

1 3



Let eqⁿ of ellipse is

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1 \quad \text{where } a > b$$

$$\Rightarrow \frac{(2)^2}{a^2} + \frac{(3)^2}{b^2} = 1$$

$$\Rightarrow \frac{4}{a^2} + \frac{9}{b^2} = 1$$

$$\frac{36}{a^2} + \frac{1}{b^2} = 1$$

$$\text{Put } \frac{1}{a^2} = u, \quad \frac{1}{b^2} = v$$

$$9(4u + 9v) = 1$$

$$36u + 81v = 1$$

$$36u + 81v = 9$$

$$\underline{\quad \quad \quad}$$

$$\underline{\quad \quad \quad}$$

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$$\Rightarrow a^2 = \frac{1}{u} = 40$$

$$\text{and } b^2 = \frac{1}{v} = 10$$

\Rightarrow Eqⁿ of ellipse is

$$\boxed{\frac{y^2}{40} + \frac{x^2}{10} = 1}$$

Section-D.

$$f(x) = \frac{\cos x}{x}$$

$$\text{so, } f(x+h) = \frac{\cos(x+h)}{x+h}$$

$$\text{now } f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{\cos(x+h)}{x+h} - \frac{\cos x}{x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x \cos(x+h) - (x+h) \cos x}{x(x+h)h}$$

$$= \lim_{h \rightarrow 0} \frac{x(\cos x \cosh - \sinh \sin x) - x \cos x - h \cos x}{x(x+h)h}$$

$$= \lim_{h \rightarrow 0} \frac{x \cos x (\cosh - 1) - x \sinh \sin x - h \cos x}{x(x+h)h}$$

033

3

1/2

1/2

1

$$= \lim_{h \rightarrow 0} \frac{x \cos x (\cosh h - 1)}{x(x+h)h} - \lim_{h \rightarrow 0} \frac{x \sin x \sinh h}{x(x+h)h}$$

$$= \lim_{h \rightarrow 0} \frac{x \cos x}{x(x+h)h}$$

We know $\left(\lim_{h \rightarrow 0} \frac{\cosh h - 1}{h} \right) = 0$, $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$= 0 \times 1 - \frac{\sin x}{x} - \frac{\cos x}{x(x+0)}$$

$$= -\frac{\sin x}{x} - \frac{\cos x}{x^2}$$

$$\Rightarrow \boxed{f'(x) = -\frac{\sin x}{x} - \frac{\cos x}{x^2}}$$

OR

$$y = \frac{4a x + 5 \sin x}{3b x + 7 \cos x}$$

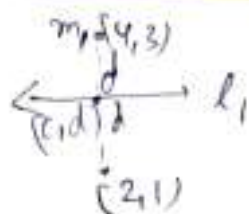
$$\frac{dy}{dx} = \frac{(3bx + 7 \cos x) \frac{d}{dx}(4ax + 5 \sin x) - (4ax + 5 \sin x) \times \frac{d}{dx}(3bx + 7 \cos x)}{(3bx + 7 \cos x)^2}$$

$$= \frac{(3bx + 7 \cos x)(4a + 5 \cos x) - (4ax + 5 \sin x)(3b - 7 \sin x)}{(3bx + 7 \cos x)^2}$$

$$= \frac{12abx + 15b \cos x + 28a \cos x + 35 \cos^2 x - [12abx - 28ax \sin x + 15b \sin x - 35 \sin^2 x]}{(3bx + 7 \cos x)^2}$$

$$= \frac{35(\cos^2 x + \sin^2 x) + (28a \cos x + 28ax \sin x) + 15b \cos x - 15b \sin x}{(3bx + 7 \cos x)^2}$$

$$= \frac{35 + 28a \cos x + 28ax \sin x - 15b \sin x + 15b \cos x}{(3bx + 7 \cos x)^2}$$



Let eqn of line l_1 is $ax + by + c = 0$

Slope of line m_1 is $\frac{1-3}{2-4} = \frac{-2}{-2} = 1$

Now line l_1 and m_1 are \perp

So, Slope of line $l_1 \times$ slope of line $m_1 = -1$

$$\Rightarrow -\frac{a}{b} \times 1 = -1$$

$$\Rightarrow -a = -b$$

$$\Rightarrow \boxed{a = b}$$

Eqn of line m_1 is $y - 3 = 1(x - 4)$

$$y - 3 = x - 4$$

$$y - x - 3 + 4 = 0$$

$$\Rightarrow \boxed{y - x + 1 = 0}$$

Let (c, d) is the point of intersection of l_1 and m_1 ,
 Also (c, d) is mid point of line m_1 ,

So, $c = \frac{4+2}{2}$, $d = \frac{3+1}{2}$

$$c = \frac{6}{2}, d = \frac{4}{2}$$

$$c = 3, d = 2$$

$\Rightarrow (3, 2)$ is passing point of line l
 and slope is $-\frac{a}{b} = -\frac{a}{a} = -1$

So, Eqn of line l_1 is

$$y - 2 = -1(x - 3)$$

$$y - 2 = -x + 3$$

$$\boxed{y + x - 5 = 0}$$

$$\boxed{y + x - 5 = 0}$$

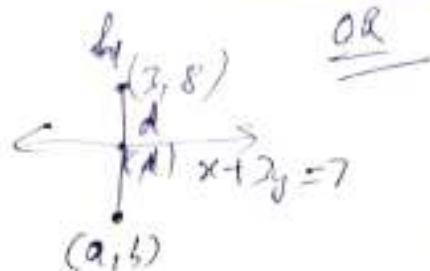
Distance $d = \frac{14}{\sqrt{2}}$
 $3x + 2y + k = 0$
 $\boxed{2x + y + \frac{k}{2} = 0}$

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

$$\frac{14}{\sqrt{2}} = \frac{|5 - \frac{k}{2}|}{\sqrt{1+1}}$$

$$\frac{14}{\sqrt{2}} = \frac{5 + \frac{k}{2}}{\sqrt{2}} = \frac{15 + k}{\sqrt{2}}$$

$$42\sqrt{2} = 15\sqrt{2} + \sqrt{2}k \Rightarrow \boxed{k = \frac{42\sqrt{2} - 15\sqrt{2}}{\sqrt{2}}}$$



OR

Let (c, d) is point of intersection

Now (c, d) is mid point of line l_1

$$\Rightarrow (c, d) = \left(\frac{3+a}{2}, \frac{8+b}{2} \right)$$

Slope of line $x+3y=7$ is $-\frac{1}{3}$

Slope of line l_1 is $\frac{b-8}{a-3}$

Now $m_1 m_2 = -1$

$$\Rightarrow \frac{1}{3} \left(\frac{b-8}{a-3} \right) = +1$$

$$\Rightarrow b-8 = 3a-9$$

$$\Rightarrow b-3a-8+9=0$$

$$\Rightarrow b-3a+1=0$$

$$\Rightarrow \boxed{3a-b-1=0}$$

$$\begin{aligned} \text{distance } d \text{ is} &= \frac{|3+3 \times 8-7|}{\sqrt{1+9}} \\ &= \frac{|3+24-7|}{\sqrt{10}} \\ &= \frac{20}{\sqrt{10}} \end{aligned}$$

$$d = \frac{|a+3b-7|}{\sqrt{1+9}} = \frac{|a+3b-7|}{\sqrt{10}}$$

$$\Rightarrow \frac{a+3b-7}{\sqrt{10}} = \frac{20}{\sqrt{10}}$$

$$\Rightarrow a+3b=27$$

$$3(a-b)=1$$

$$9a-3b=3$$

$$\frac{10a=30}{a=3}$$

$$b=8$$

$$a+3b-7=-20$$

$$a+3b=-20+7$$

$$3(a+b)=-13$$

$$3a-b=1$$

$$3a+9b=-39$$

$$\frac{10b=40}{b=4}$$

$$3a=-1-4$$

$$3a=-5$$

$$a=-\frac{5}{3}$$

$$\Rightarrow \text{mirror image} = \left(-\frac{5}{3}, -4 \right)$$

(34)

$$\begin{aligned}
 A &= \{x: x \in \mathbb{R}, x^2 - 8x + 12 = 0\} \\
 &= \{x: x \in \mathbb{R}, x^2 - 6x - 2x + 12 = 0\} \\
 &= \{x: x \in \mathbb{R}, (x-6)(x-2) = 0\} \\
 &= \{6, 2\}
 \end{aligned}$$

$$\begin{aligned}
 B &= \{x: x \in \mathbb{R}, 2 \leq x \leq 7\} \\
 &= [2, 7]
 \end{aligned}$$

$$(i) A \cap B = \{2, 6\}$$

$$(ii) A - B = \emptyset$$

$$(iii) A' \cap B = B - A = (2, 6) \cup (6, 7]$$

Q35

$$(1+2x)^6 (1-x)^7$$

$$(1+x)^n = nC_0 + nC_1 x + nC_2 x^2 + nC_3 x^3 + \dots + nC_n x^n$$

$$\text{So, } (1+2x)^6 = 6C_0 + 6C_1 (2x) + 6C_2 (2x)^2 + \dots + 6C_6 (2x)^6$$

$$(1-x)^7 = 7C_0 + 7C_1 (-x) + 7C_2 (-x)^2 + \dots + 7C_7 (-x)^7$$

$$\text{Now } (1+2x)^6 (1-x)^7 = \{6C_0 + 6C_1 (2x) + 6C_2 (2x)^2 + \dots + 6C_6 (2x)^6\} \{7C_0 + 7C_1 (-x) + 7C_2 (-x)^2 + \dots + 7C_7 (-x)^7\}$$

$$\text{To find Coefficient of } x^5 \text{ are } [6C_5 (2x)^5 7C_0 + 6C_4 (2x)^4 7C_1 (-x) + 6C_3 (2x)^3 7C_2 (-x)^2 + 6C_2 (2x)^2 7C_3 (-x)^3 + 6C_1 (2x) 7C_4 (-x)^4 + 6C_0 (2x)^0 7C_5 (-x)^5]$$

$$\text{Coefficients of } x^5 = (2)^5 6C_5 7C_0 - 6C_4 (2)^4 7C_1 + 6C_3 (2)^3 7C_2 - 6C_2 (2)^2 7C_3 + 6C_1 (2) 7C_4 - 6C_0 7C_5$$

$$= 32 \times \frac{6!}{5!1!} \times \frac{7!}{0!7!} - \frac{6!}{4!2!} (2)^4 \times \frac{7!}{1!6!} +$$

$$\frac{6!}{3!3!} \times 8 \times \frac{7!}{2!5!} - \frac{6!}{2!4!} \times 4 \times \frac{7!}{3!4!} + \frac{6!}{1!5!} \times 2 \times \frac{7!}{4!3!} - \frac{6!}{0!6!} \times \frac{7!}{5!2!}$$

$$= 32 \times 6 - 6 \times 5 \times 8 \times 7 + \frac{6 \times 5 \times 4 \times 8 \times 7 \times 6 \times 5}{3 \times 2} - \frac{6 \times 5 \times 4 \times 7 \times 6 \times 5}{2 \times 2} + 6 \times 2 \times \frac{7 \times 6 \times 5}{3 \times 2} - \frac{7 \times 6 \times 5}{2}$$

$$= 32 \times 6 - 6 \times 5 \times 56 + 20 \times 8 \times 21 - 10 \times 7 \times 30 + 14 \times 30 - 21$$

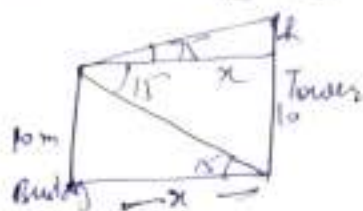
$$= 192 - 1680 + 3360 - 2100 + 420 - 21$$

$$= 171$$

Section - E

Q36 (i) $\tan 15 = \tan(45-30) = \frac{1-\tan 30}{1+\tan 30} = \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

(ii)



$$\frac{10}{x} = \tan 15 = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$x = \frac{10(\sqrt{3}+1)}{\sqrt{3}-1}$$

(iii) $\cos 75 = \cos(45+30)$
 $= \cos 45 \cos 30 - \sin 45 \sin 30$
 $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$
 $= \frac{\sqrt{3}-1}{2\sqrt{2}}$

$$\frac{h}{x} = \tan 75 = \frac{\text{OR}}{\tan(90-15)} = \cot 15 = \frac{1}{\tan 15}$$

$$\frac{h}{x} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$h = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{10(\sqrt{3}+1)}{\sqrt{3}-1} = \frac{10(3+1+2\sqrt{3})}{3-1}$$

$$= \frac{10(4+2\sqrt{3})}{2} = 5(4+2\sqrt{3}) = 20+10\sqrt{3}$$

Total candidates = 5

No. of males = 3

No. of females = 2

(i) Probability that both are males = $\frac{{}^3C_2}{{}^5C_2}$
 $= \frac{\frac{3!}{2!1!}}{\frac{5!}{2!3!}} = \frac{3!}{2!} \times \frac{2!3!}{5!}$
 $= \frac{3 \times 2 \times 2 \times 2}{5 \times 4 \times 3 \times 2}$
 $= \frac{3}{10}$

(ii) Probability that both are female = $\frac{{}^2C_2}{{}^5C_2}$
 $= \frac{1}{\frac{5!}{2!3!}} = \frac{2!3!}{5!}$
 $= \frac{1}{10}$

(iii) Probability that one is male and one is female = $\frac{{}^3C_1 \times {}^2C_1}{{}^5C_2} = \frac{3 \times 2}{\frac{5!}{2!3!}} = \frac{3 \times 2 \times 2 \times 2}{5 \times 4 \times 3 \times 2}$
 $= \frac{3}{5}$

(38)

$$3, 3^2, 3^3, \dots$$

(i) It is GP with $a=3, r=3$

$$\text{So, } a_5 = ar^4 = 3 \times (3)^4 = 3^5$$

(ii) $S_n = \frac{a(r^n - 1)}{r - 1}$

$$S_5 = \frac{a(r^5 - 1)}{r - 1} = \frac{3(3^5 - 1)}{3 - 1} = \frac{3}{2}(3^5 - 1)$$

(iii) $a_n = 6561$

$$a_n = ar^{n-1}$$

$$6561 = 3 \times (3)^{n-1} = 3^n$$

$$(3) = 3^n$$

$$\boxed{n=8} \text{ Ans.}$$

So, 6561 persons will be helped on 8th day.

$$\begin{array}{r} 3 \overline{) 6561} \\ \underline{2187} \\ 9 \\ \underline{729} \\ 9 \\ \underline{81} \\ 9 \end{array}$$

2

4

Note:- Any relevant solution not given herein but done by the students is suitably awarded

Q.No.	Value Points / Key Points	Value Point	Total Point
1	d) $(\frac{1}{4}, 0); \frac{1}{4}$	1	
2	d) $\text{INF} = \phi$	1	
3	b) $\frac{y^2}{9} - \frac{x^2}{7} = 1$	1	
4	c) 25	1	
5	d) $\pi/4$	1	
6	b) $3/2$	1	
7	c) $\sqrt{3}/2$	1	
8	d) 6	1	
9	a) 26	1	
10	a) $E(1, 2)$	1	
11	d) 6	1	
12	a) $\sqrt{66}$	1	
13	b) $0 + \frac{1}{2}$	1	
14	b) 2	1	
15	a) $(-1, 0, 4)$	1	
16	d) $\{1, 2, 3, 4\}$	1	
17	c) $-x^2 \sin x + 3x \cos x$	1	
18	b) 0.2	1	
19	a) Both A and R are true and R is the correct explanation of A	1	
20	d) A is false and R is true.	1	

Q21

Section-B

$$f(x) = 2x^2 + 3x - 5, \quad g(x) = x - 1$$

$$\begin{aligned} \frac{f}{g}(x) &= \frac{f(x)}{g(x)} = \frac{2x^2 + 3x - 5}{x - 1} = \frac{2x^2 + 5x - 2x - 5}{x - 1} \\ &= \frac{x(2x + 5) - 1(2x + 5)}{x - 1} \\ &= \frac{(2x + 5)(x - 1)}{x - 1} = 2x + 5 \end{aligned}$$

$$\Rightarrow \frac{f}{g}(x) = 2x + 5$$

$$\text{Domain of } \frac{f}{g}(x) = \text{Domain of } \frac{2x^2 + 3x - 5}{x - 1} = \mathbb{R} - \{1\}$$

$$\text{Range of } \frac{f}{g}(x) = \dots$$

$$y = 2x + 5$$

$$2x = y - 5$$

$$x = \frac{y - 5}{2}$$

$$\text{Here } x \neq 1 \Rightarrow \frac{y - 5}{2} \neq 1$$

$$y \neq 7$$

$$\Rightarrow \text{Range of } \frac{f}{g}(x) = \mathbb{R} - \{7\}$$

Q22

$$AB = \sqrt{(1-0)^2 + (1-7)^2 + (-6+1)^2}$$

$$= \sqrt{1+1+16} = \sqrt{18}$$

$$BC = \sqrt{(4-1)^2 + (9-0)^2 + (-6+6)^2}$$

$$= \sqrt{9+9} = \sqrt{18}$$

$$AC = \sqrt{(4-0)^2 + (9-7)^2 + (-6+1)^2}$$

$$= \sqrt{16+4+16} = \sqrt{36} = 6$$

Here $AB = BC$, so, A, B, C are vertices of isosceles Δ .

Q23

$$\text{L.H.L. } \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x^2 - 1 = \lim_{h \rightarrow 0} (1-h)^2 - 1 = 1 - 1 = 0$$

$$\text{R.H.L. } \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} -x^2 - 1 = \lim_{h \rightarrow 0} -(1+h)^2 - 1 = -2$$

so, L.H.L. \neq R.H.L.

so, Limit does not exist

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin ax + bx}{ax + \sin bx} &= \lim_{x \rightarrow 0} \left(\frac{\sin ax}{ax} \times ax \right) + \lim_{x \rightarrow 0} bx \\ &= \lim_{x \rightarrow 0} (ax + bx) = \lim_{x \rightarrow 0} (a + b)x = 0 \end{aligned}$$

29

$$\frac{x^2}{4} + \frac{y^2}{36} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a^2 = 36, \quad b^2 = 4$$

$$c^2 = a^2 - b^2 = 36 - 4 = 32$$

$$c^2 = 32$$

$$c = \pm 4\sqrt{2}$$

$$\text{foci} = (0, \pm c) \\ = (0, \pm 4\sqrt{2})$$

$$\text{vertices} = (0, \pm a) = (0, \pm 6)$$

$$e = \frac{c}{a} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$$

$$\text{length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times 4}{6} = \frac{4}{3}$$

25

$$(a-2)^2 + (0-2)^2 = 25$$

OR

$$(a-2)^2 + (0-2)^2 = 25$$

$$(a-2)^2 = 25 - 4 = 16$$

$$a^2 + 4 - 4a = 16$$

$$a^2 - 4a - 12 = 0$$

$$a^2 - 6a + 2a - 12 = 0$$

$$(a-6)(a+2) = 0$$

$$a = -2, 6$$

Case I

$$\text{Centre} = (-2, 0)$$

Eqⁿ of Circle

$$(x+2)^2 + y^2 = 25$$

$$x^2 + y^2 + 4x - 21 = 0$$

Case II Centre (6, 0)

Eqⁿ

$$(x-6)^2 + y^2 = 25$$

$$x^2 + y^2 - 12x + 11 = 0$$

$$z_1 z_2 = (2-i)(1+i) = 2 + 2i - i - 1 = 1 + i$$

$$\frac{z_1 z_2}{z_1} = \frac{2+i}{2-i} \times \frac{2-i}{2-i} = \frac{6-3i+2i-1}{4-1} = \frac{5-i}{3}$$

$$= \frac{5-i}{3}$$

$$\text{Re}\left(\frac{z_1 z_2}{z_1}\right) = \frac{5}{3}$$

Section-C

$$y = \frac{x^2 \cos x}{\sin x} = \frac{x^2}{\sin x}$$

$$\frac{dy}{dx} = \frac{1}{\sin^2 x} \left[\frac{\sin x \times 2x - x^2 \cos x}{\sin^2 x} \right]$$

$$= \frac{2x \sin x - x^2 \cos x}{\sin^3 x}$$

 $\frac{1}{2} \text{ each}$

1

 $\frac{1}{2}$ $\frac{1}{2}$

2

 $\frac{1}{2}$ $\frac{1}{2}$

2

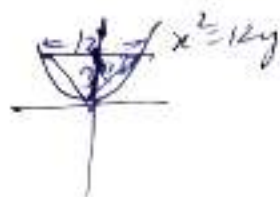
 $\frac{1}{2}$

2

 $\frac{1}{2}$

3

26



$$x^2 = 4ay$$

$$4a = 12$$

$$a = 3 \quad \text{focus} = (0, a) = (0, 3)$$

$$\text{Area} = \frac{1}{2} \times b \times h = \frac{1}{2} \times 12 \times 3 = 18$$

OR

$$\text{foci} = (\pm 4, 0) = (\pm c, 0) \quad [c = 4]$$

$$\frac{2b^2}{a} = 12$$

$$[2b^2 = 12a]$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$c^2 = a^2 + b^2$$

$$16 = a^2 + b^2$$

$$\text{When } a = -8$$

$$b^2 = 60 = 6 \times 10$$

$$= -48$$

rejected

$$\text{Also when } a = 2$$

$$b^2 = 60 = 12$$

$$E \Rightarrow$$

$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$

$$a^2 + 60 - 16 = 0$$

$$a^2 + 80 - 2a - 16 = 0$$

$$(a+8)(a-2) = 0$$

$$a = -8, 2$$

(39)

9B, 4G (No. of) ways when committee has at least 3 girls

$$= {}^3C_6 4B + 4G 3B$$

$$= {}^4C_2 \times {}^9C_4 + {}^4C_4 \times {}^9C_3$$

$$= 504 + 84$$

$$= 588$$

(50)

$$37 - (3n + 5) \geq 9n - 8(n - 3)$$

$$37 - 3n - 5 \geq 9n - 8n + 24$$

$$32 - 24 \geq n + 3n$$

$$8 \geq 4n$$

$$n \leq 2 \Rightarrow n \in \{0, 1, 2\}$$



21

or

$$\cos x = -\frac{1}{3}$$

$$180^\circ \leq x \leq 270^\circ$$

$$90^\circ \leq \frac{x}{2} \leq 135^\circ$$

$\Rightarrow \frac{x}{2}$ lies in IInd quadrant

$$\therefore \sin \frac{x}{2} = \frac{1 - \cos x}{2}$$

$$= \frac{1 + \frac{1}{3}}{2} = \frac{\frac{4}{3} \times \frac{1}{2}}{1} = \frac{2}{3}$$

$$\sin \frac{x}{2} = \pm \sqrt{\frac{2}{3}}$$

$\Rightarrow \sin \frac{x}{2} = \sqrt{\frac{2}{3}}$ as $\frac{x}{2}$ lies in IInd quadrant

(32)

OR

$$y = \frac{3ax + 5\cos x}{4bx + 7\sin x}$$

$$\frac{dy}{dx} = \frac{(4bx + 7\sin x)(3a - 5\sin x) - (3ax + 5\cos x)(4b + 7\cos x)}{(4bx + 7\sin x)^2}$$

$$= \frac{(4bx + 7\sin x)(3a - 5\sin x) - (3ax + 5\cos x)(4b + 7\cos x)}{(4bx + 7\sin x)^2}$$

$$= \frac{(12bxa - 20bx\sin x + 21a\sin x - 35\sin^2 x) - (120bx + 21ax\cos x + 20b\cos x + 35\cos^2 x)}{(4bx + 7\sin x)^2}$$

$$= \frac{-20bx\sin x + 21a\sin x - 35\sin^2 x - 21ax\cos x - 20b\cos x - 35\cos^2 x}{(4bx + 7\sin x)^2}$$

$$= \frac{21a\sin x - 20bx\sin x - 21ax\cos x - 35 - 20b\cos x}{(4bx + 7\sin x)^2}$$

$$= \frac{21a(\sin x - x\cos x) - 20b(x\sin x + \cos x) - 35}{(4bx + 7\sin x)^2}$$