O. S. D. A. V. Public School, Kaithal.
December Exam. 2024-2025
Class : XII
Subject: Mathematics (Applied) **Set-A**

Time 3 hrs.
M.M. 80

General Instructions:

1. This question paper contains **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions
2. **Section A** has **18 MCQ's** and **02 Assertion-Reason based** questions of 1 mark each.
3. **Section B** has **5 Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C has 6 Short Answer (SA)-type** questions of 3 marks each.
5. **Section D has 4 Long Answer (LA)-type** questions of 5 marks each.
6. **Section E has 3 source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

Section A

1. In 1 km race, player P beats player Q by 18m or 9 sec. What is P's time to complete the race?

   a. 512 sec. b. 502 sc. c. 491 sec. d. 481 sec.

2. The values of x satisfying $\frac{1}{2}\left(\frac{3}{5}x + 4\right) \geq \frac{1}{3}(x - 6)$ are:

   a. $x \geq 120$ b. $x \leq 120$ c. $x \leq 12$ d. none of these

3. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is:

   a. 3 b. 0 c. -1 d. 1

4. If AB = A and BA = B, then B^2 equals :

   a. 0 b. I c. A d. B

5. If $y = e^{-2x}$, then $\frac{d^3y}{dx^3}$ is equal to:

   a. $2e^{-2x}$ b. e^{-4x} c. $4e^{-4x}$ d. $-8e^{-2x}$

6. Find the interval in which the function $f(x) = x^2 - 2x$ is strictly increasing.

   a. $[1, \infty)$ b. $(1, \infty)$ c. $(0, \infty)$ d. $-\infty, 1)$

7. The order and the degree of the differential equation $5\frac{d^2y}{dx^2} = \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{1/4}$ are respectively

   a. 2, 2 b. 1, 2 c. 2, 1 d. 2, 4

8. A bag contains 2 white and 4 black balls. A ball is drawn 5 times with replacement. The probability that at least 4 of the balls drawn are white is:

   a. $\frac{8}{243}$ b. $\frac{10}{243}$ c. $\frac{11}{243}$ d. $\frac{32}{243}$

9. If m is the mean of a poison distribution, the variance is given by:
   a. m^2 b. $m^{1/2}$ c. m d. $\frac{m}{2}$
10. What time will it be after 800 hours, if the present time is 7:00 pm?
   a. 3pm b. 3am c. 4am d. 4pm
11. If the calculated value of $|t| < t_v(\alpha)$ (critical value of t), then the null hypothesis
   a. is rejected b. is accepted
   c. is neither accepted nor rejected d. cannot be determined.
12. In a one-sample t-test, the degrees of freedom are calculated as:
   a. n – 1, where n is the sample size. b. n, where n is the sample size.
   c. n + 1, where n is the sample size. d. $\frac{n}{2}$, where n is the sample size.
13. For the given values 15, 24, 18, 33, 42, the 3 yearly moving averages are:
   a. 19, 22, 33 b. 19, 25, 31 c. 19, 30, 31 d. 19, 22, 30
14. What sum of money should be deposited at the end of every 6 months to accumulate Rs. 50000 in 8 yr, if money is worth 6% per annum compounded semi-annually? [Given : (1.03)16 = 1.6047]
   a. Rs. 3432.53 b. Rs. 2783.08 c. Rs. 2480.57 d. Rs. 2149.93
15. The annual depreciation of a car is Rs. 30000, if the scrap value of the car after 15 yr. is Rs. 50000, then original cost of the car, when depreciation is linear.
   a. Rs. 600000 b. Rs. 450000 c. Rs. 495000 d. Rs. 500000
16. Using flat rate method, the EMI to repay a loan of Rs. 500000 by equal monthly payments in 10 yr at an interest rate of 7.5% p.a. compounded monthly is:
   a. Rs. 5935 b. Rs. 6380 c. Rs. 7340 d. Rs. 8520
17. The graph of the inequation $2x + 3y > 6$ is the :
   a. entire XOY – plane
   b half-plane that neither contains the origin nor the points on the line $2x + 3y = 6$
   c. half-plane that contains the origin.
   d. whole XOY – plane excluding the points on the line $2x + 3y = 6$
18. The maximum value of Z for the problem maximize $Z = 2x + y$ subject to constraints $x + y \leq 2$, $x \geq 0$, $y \geq 0$ is:
   a. 4 b. 3 c. 1 d. 0

Assertion-Reason Based questions:

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion: (A) If x is real, then the minimum value of $x^2 - 8x + 17$ is 1.

Reason: (R) If f ″ (x) > 0 at critical point, then the value of the function at critical point will be the minimum value of the function.

20. In a binomial distribution n = 200 and p = 0.04. Taking poisson distribution as an approximation to the binomial distribution.

Assertion (A) Mean of Poisson distribution = 8

Reason (R) P (X = 4) = $\frac{512}{3e^8}$.

Section B

21. a. Using the properties of determinants prove that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$ or

b. Using properties of determinants to solve for x;

$$\begin{vmatrix} x+a & b & c \\ c & x+b & a \\ a & b & x+c \end{vmatrix} = 0 \text{ and } x \neq 0$$

22. Solve the following system of equations using Cramer's rules.

2x – y = 17

3x + 5y = 6

23. Find the solution to the following LPP (if it exists) graphically:

Maximize Z = 50x + 30y

Subject to the constraints $2x + y \leq 18$, $3x + 2y \leq 34$ and $x, y \geq 0$

24. A money lender charges interest at the rate of 5 paise per rupee per month, payable in advance. What effective rate of interest does he charge per month?

25. a. The present value of a perpetual income of Rs. x at the end of each 6 months is Rs. 144000. Find the value of x if money is worth 6% compounded semi-annually.

or

b. Mrs. Sarita purchases 100 shares of a company that cost Rs. 250 each. After one the price of each share rise to Rs. 300. Assuming that there no trading cost and no dividends, find the nominal rate of return on the investment.

Section C

26. A bottle is full of dettol. One third of it is taken out and then an equal amount of water is poured into the bottle to fill it. This operation is repeated four times. Find the final ratio of dettol and water in the bottle.

27. a. Sketch the region bounded by $y = 2x - x^2$ and X-axis and find its area using integration. or

b. Evaluate: $\int_0^1 x \log(1 + 2x)dx$

28. Find the differential equation of the family of circles having their centres on X-axis.

29. Find the probability distribution of the number of sixes in three throws of a die. Also find the mean of the distribution.

30. The mean weekly sales of soap bars in departmental stores were 146.3 bars per store. After an advertizing campaign the mean weekly sales in 400 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful?

31. a. Find the effective rate of return equivalent to declare rate of 12% compounded.

i. semi-annually ii. quarterly iii. monthly

or

b. The cost of a washing machine depreciates by Rs. 720 during the second year ad by Rs. 648 during the third year. Calculate:

i. the rate of depreciation per annum.

ii. the value of machine at the end of third year.

Section D

32. a. Find the intervals in which the following function f is strictly increasing or strictly decreasing: $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ or

b. Find all the points of local maxima and local minima of the function:

$$f(x) = 2x^3 - 21x^2 + 36x - 20$$

33. a. The probability distribution of a random variable X is given as under:

$$P(X=x) = \begin{cases} kx^2 & for\ x = 1, 2, 3 \\ 2kx, & for\ x = 4, 5, 6 \\ 0, & otherwise \end{cases}$$ where k is a constant.

i. What is he value of k?

ii. Find the probability P(X < 4).

iii. Find the probability P (x ≥ 4).

or

b. An airline accepts reservation for the seats on a particular flight of a 98 seater aircraft. I is known from past experience that 3% of the persons who reserve seats do not turn up and so the airline has a policy of a allowing 100 persons to book seats on the flight. What is the probability of more than 98 persons turning up for the flights?

34. Three pipes A, B and C connected to a tank. Out of the three A and B are inlet pipes and C is outlet pipe. If opened separately, A fills the tank in 10 hours, B fills the tank in 12 hours. If all three are opened simultaneously, how much time does it take to fill the tank?

35. Aman buys a car for which he makes down payment of Rs. 150000 and the balance is to be paid in 2yr by monthly installment of Rs. 25448 each. If he financer charges interest at the rate of 20% p.a., find the actual price of the car.

(Given $\left(\frac{61}{60}\right)^{-24} = 0.67252$)

Section E

36. On her birthday, Prema decides to donate some money to children of an orphanage home.

If there are 8 children less, everyone gets Rs. 10 more.

However, if there are 16 children more, everyone gets Rs. 10 less.

Let the number of children in the orphanage home be x and the amount to be donated to each child be Rs. y.

Based on the above information, answer the following questions:

i. Write the system of linear equations in x and y formed of the given situation.

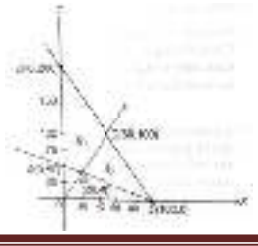
ii. Write the system of linear equations, obtained in (i) above, in matrix form AX=B.

iii. a. Find the inverse of matrix A. OR b. Determine the values of x and y.

37. In number theory, it is often important to find factors of an integer N.

The number N has two trivial factors, namely 1 ad N. Any other factor, if exists, is called non-trivial factor of N.

Naresh has plotted a graph of some constraints (linear in equations) with points A(0,50), B(20,40)



C(50,100), D(0,200) and E(100,0). This graph is
constructed using three non-trivial constraints and two trivial constraints. One of the non-rival constraints is $x+2y \geq 100$

Based on the above information, answer the following questions:

i. What are the two trivial constraints?

ii. a. If R_1 is the feasible region, then what are the other two non-trivial constraints? or

b. If R_2 is the feasible region, then what are the other two non-trivial constraints?

iii. If R_1 is the feasible region, then find he maximum value of the objective function $Z=5x+2y$.

38. When observed over a long period of time, a time series data can predict trends that can forecast increase or decrease or stagnation of a variable under consideration. Such analytical studies can benefit a business for forecasting or prediction of future estimated sales or production.

The table below shows the sale of an item in a district during 1996-2001

Year	1996	1997	1998	1999	2000	2001
Sales (in lakh Rs.)	6.5	5.3	43	6.1	5.6	7.8

Based on the above information answer the following questions:

i. Determine the equation of the straight-line trend.

ii. a. Tabulate the trend values of the years and also compute expected sales trend for the year 2002.

or

b. Fit a straight-line trend by the method of least squares for the following data

Year	2004	2005	2006	2007	2008	2009	2010
Profit (Rs. 100)	114	130	126	144	138	156	164

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Section A

1. In a 10 km race, P, Q and R, each running at uniform speed, get the gold, silver and bronze medals, respectively. If P beats Q by 1 km and Q beats R by 1 km, then by how many meters does P beats R?

   a. 1600 b 1500 c. 1400 d. 1900

2. If $x \in R$, $|x| \geq -6$, then:

   a. $x \in (-\infty, -6) \cup (6, \infty)$ b. $x \in R$

   c. $x \in [-6, 6]$ d. $x \in (-\infty, -6) \cup [6, \infty)$

3. If AB = A and BA = B, then B^2 equals :

   a. 0 b. I c. A d. B

4. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is:

   a. 3 b. 0 c. -1 d. 1

5. The function $f(x) = x^2e^{-x}$ strictly increases on:

   a. [0, 2] b. [0, ∞) c. (-∞, 0] ∪[2, ∞) d. None of these

6. If $y = e^{-2x}$, then $\frac{d^3y}{dx^3}$ is equal to:

   a. $2e^{-2x}$ b. e^{-4x} c. $4e^{-4x}$ d. $-8e^{-2x}$

7. The order and degree of the differential equation:

   $\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = 2x^2 \log\left(\frac{d^2y}{dx^2}\right)$ are respectively

   a. 1, 1 b. 2, 1 c. 1, 2 d. 2, not defined

8. If m is the mean of a poison distribution, the variance is given by:
   a. m^2 b. $m^{1/2}$ c. m d. $\frac{m}{2}$
9. What time will it be after 800 hours, if the present time is 7:00 pm?
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Assertion (A) Mean of Poisson distribution = 8

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21. a. Using the properties of determinants prove that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$ or

b. Using properties of determinants to solve for x;

$$\begin{vmatrix} x+a & b & c \\ c & x+b & a \\ a & b & x+c \end{vmatrix} = 0 \text{ and } x \neq 0$$

22. At 5% p.a. compounded yearly, find the present value of a perpetuity of Rs. 900 payable at the end of each year.

23. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find the value of k such that $A^2 - kA + 2I = 0$

24. Find the solution to the following LPP (if it exists) graphically:

Maximize Z = 50x + 30y

Subject to the constraints $2x + y \leq 18$, $3x + 2y \leq 34$ and $x, y \geq 0$

25. a. The present value of a perpetual income of Rs. x at the end of each 6 months is Rs. 144000. Find the value of x if money is worth 6% compounded semi-annually.

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35. Aman buys a car for which he makes down payment of Rs. 150000 and the balance is to be paid in 2yr by monthly installment of Rs. 25448 each. If he financer charges interest at the rate of 20% p.a., find the actual price of the car.

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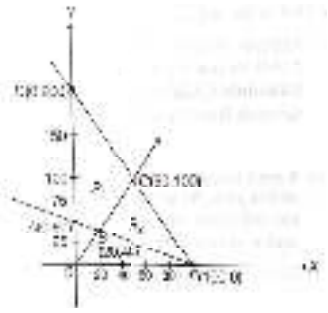
Naresh has plotted a graph of some constraints (linear in equations) with points A(0,50), B(20,40) C(50,100), D(0,200) and E(100,0). This graph is constructed using three non-trivial constraints and two trivial constraints. One of the non-rival constraints is $x+2y \geq 100$



Based on the above information, answer the following questions:

i. What are the two trivial constraints?

ii. a. If R_1 is the feasible region, then what are the other two non-trivial constraints? or

b. If R_2 is the feasible region, then what are the other two non-trivial constraints?

iii. If R_1 is the feasible region, then find he maximum value of the objective function $Z=5x+2y$.

December Examination 2024-25
Mathematics (Applied)
Class : XII
Set A and B

Marking Scheme / Hints to Solution

Note :- Any relevant Solution not given here in but done by students will be suitably awarded.

No.	Section A Value Points / Key Points		Total Point
1	(c) 491 Sec.	1	1
2	(b) $x \leq 120$	1	1
3 (B)	(c) -1	1	1
4	(d) B		
5	(d) $-8e^{-2x}$	1	1
6	(b) $(1, \infty)$	1	1
7.	(d) 2, 4	1	1
8	(c) $\frac{11}{243}$	1	1
9	(c) m	1	1
10	(b) 3 a.m.	1	1
11	(b) is accepted	1	1

13 12(B)	(b) 19, 25, 31	1	1
14 13(B)	(c) Rs. 2480.57	1	1
15 16(B)	(d) Rs. 500000	1	1
16 15(B)	(a) Rs. 5935	1	1
17 18(B)	(b) half-plane that neither contains the origin nor the points on the line $2x+3y=6$	1	1
18 17(B)	(a) 4	1	1
19 19(B)	(a) Both (A) and (R) are true and (R) is the correct explanation of (A).	1	1
20 20(B)	(b) Both (A) and (R) are true and (R) is not the correct explanation of (A).	1	1
	Different questions of Set B Section A		
1	(d) 1900	1	1
2.	(b) $x \in R$	1	1
5	(a) $[0, 2]$	1	1
7.	(d) 2, not defined	1	1
21 21(B)	Section B $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$		

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$ ½

$$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$ ½

$C_1 \to C_1 - C_2 \quad C_2 \to C_2 - C_3$

$$(a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b+c+a & -(a+b+c) & 2b \\ 0 & a+b+c & c-a-b \end{vmatrix}$$ ½

$$(a+b+c)^3\begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & 2b \\ 0 & 1 & c-a-b \end{vmatrix}$$ ½

Expanding by R_1

$(a+b+c)^3[1(1-0)] = (a+b+c)^3$ 2

OR

21

$$\begin{vmatrix} x+a & b & c \\ c & x+b & a \\ a & b & x+c \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & a \\ x+a+b+c & b & x+c \end{vmatrix} = 0$$ ½

$$(x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & a \\ 1 & b & x+c \end{vmatrix} = 0 \qquad 1\frac{1}{2}$$

$R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$(x+a+b+c)\begin{vmatrix} 0 & -x & c-a \\ 0 & x & a-x-c \\ 1 & b & x+c \end{vmatrix} = 0 \qquad 1\frac{1}{2}$$

$$(x+a+b+c)\left[1(-ax+x^2+cx-cx+ax)\right] = 0$$

$$x^2(x+a+b+c) = 0 \qquad 1\frac{1}{2}$$

$x \neq 0$ $\therefore$ $x+a+b+c=0$

$x = -(a+b+c)$

2

$$2x - y = 17$$
$$3x + 5y = 6$$

$$D = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix} \qquad 1\frac{1}{2}$$

$|D| = 10+3 = 13$

$$D_1 = \begin{vmatrix} 17 & -1 \\ 6 & 5 \end{vmatrix} \qquad 1\frac{1}{2}$$

$D_1 = 85 + 6 = 91$

$$D_2 = \begin{vmatrix} 2 & 17 \\ 3 & 6 \end{vmatrix} \qquad 1\frac{1}{2}$$

$D = |12 - 51| = -39$

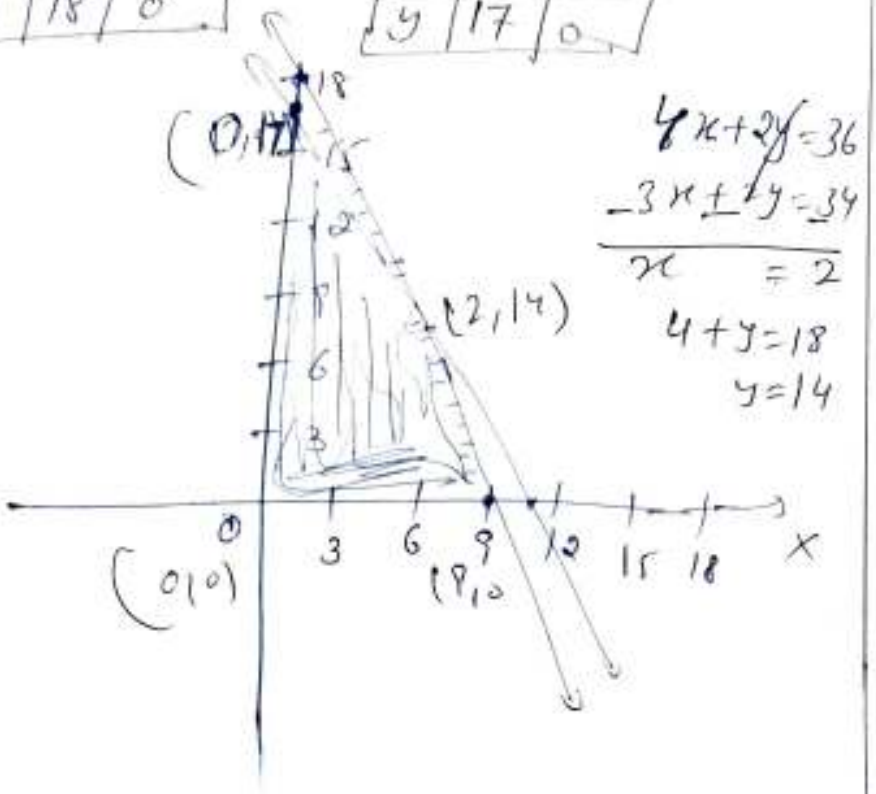
23
24(B) $2x+y=18,\ 3x+2y=34,\ x,y\geq 0$

$2x+y=18$

x	0	9
y	18	0

$3x+2y=34$

x	0	34/3
y	17	0



$4x+2y=36$

$-3x\pm y=34$

$x=2$

$4+y=18$

$y=14$

$Z=50x+30y$

$Z(0,0)=0 \qquad Z(9,0)=450$

$Z(0,17)=510 \qquad Z(2,14)=100+420=520$

Maximum

Maximum Value 520 at (2,14)

24 $r_c = \left(1 + \frac{r}{100k}\right)^k - 1$

$= \left(1 + \frac{5\cancel{0}^{1}}{\cancel{100}_{20} \times 12}\right)^{12} - 1$ — 1

$= \left(1 + \frac{1}{240}\right)^{12} - 1$

$= \left(\frac{241}{240}\right)^{12} - 1$ — 2

$= (1.004)^{12} - 1$ — 1

$= 8.6 - 1 = 8.5\%$

25 $P = ₹144000$

25(B) $P = \frac{R}{i}$ $\quad i = \frac{6}{100 \times 2}$ — 1

$144000 = \frac{x \times 200}{6}$ — 2

$\frac{144\cancel{0}^{720} \times 6}{\cancel{2}} = x$

$x = ₹4320$ — Ans — 1

(B) OR

Nominal rate $= \frac{V_f - V_i}{V_i} \times 100$ — 1

$V_i = 100 \times 250 = 25000$

$V_f = 300 \times 100 = 30000$ — 2

Nominal rate $= \frac{30000 - 25000}{25000} \times 100$

$= \frac{5000}{25000} \times 100 = 20\%$ — 1

Diff questions of set B of Section B

92

$P = ₹ 9\omega \quad R = ₹ 9\omega$

$i = \frac{5}{100}$

$P = \frac{R}{i}$ — 1½

$P = \frac{9\omega}{\frac{5}{100}} = \frac{9\omega \times 100}{5} = ₹ 18000$ — ½

2

93

$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$= \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix}$

$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$ — 1

$A^2 - kA + 2I = 0$

$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$

$\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$ — 1

$\therefore \quad k = 1$

2

Section C

26 Let dettol in bottle = x l.

A·T·D

27(B)

Amount of dettol left in bottle:-

$= x\left(1-\frac{y}{x}\right)^{n}$ — 1

$= x\left(1-\frac{x}{\frac{3}{x}}\right)^{4}$

$= x\left(\frac{2}{3}\right)^{4} = \frac{16}{81}x$ — 1 — 3

$\therefore$ water in bottle $= \frac{65}{81}x$ — 1

Req. Ratio $\frac{16}{81}x : \frac{65}{81}x$ — 1

$= 16:65$

27

28(B)

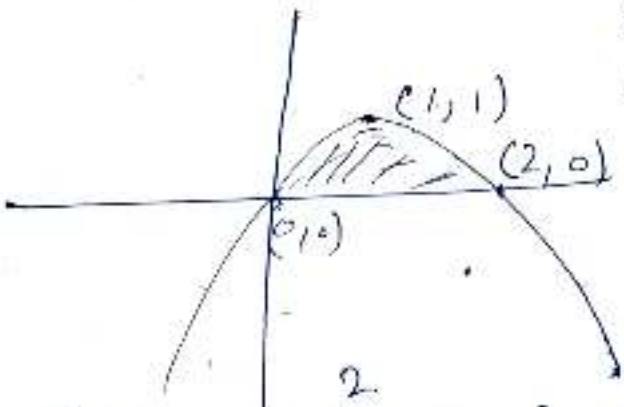
$y = 2x - x^2$

$y = 2x - x^2 + 1 - 1$

$= -(x^2 + 1 - 2x) + 1$

$y = 1 - (x-1)^2$

$(x-1)^2 = -(y-1)$



$2x - x^2 = 0$

$x(2-x) = 0$

$x = 0$

$x = 2$ — 1½

Req. Area $= \int_0^2 2x - x^2 \, dx$

$\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2$ $1\frac{1}{2}$

$4 - \frac{8}{3} = \frac{4}{3}$ sq unit

or

27 $\int_0^1 x \log(1+2x)\,dx$

II I

$\frac{x^2}{2}\log(1+2x)\Big|_0^1 - \int_0^1 \frac{2}{1+2x} \times \frac{x^2}{2}\,dx$ 1

$\frac{1}{2}\log 3 - \int_0^1 \frac{x^2}{2x+1}\,dx$

$2x+1 \overline{)x^2}\ (\frac{1}{2}x - \frac{1}{4}$

$x^2 + \frac{1}{2}x$

$-\frac{1}{2}x$

$-\frac{1}{2}x - \frac{1}{4}$

$\frac{1}{4}$

$\frac{1}{2}\log 3 - \int_0^1 \frac{1}{2}x - \frac{1}{4} + \frac{\frac{1}{4}}{2x+1}\,dx$ 1 3

$\frac{1}{2}\log 3 - \left[\frac{1}{2}\frac{x^2}{2} - \frac{1}{4}x + \frac{1}{4}\log\frac{|2x+1|}{2}\right]_0^1$

$\frac{1}{2}\log 3 - \left[\frac{1}{4} - \frac{1}{4} + \frac{1}{8}\log 3\right]$

$\frac{1}{2}\log 3 - \frac{1}{8}\log 3 = \left(\frac{1}{2} - \frac{1}{8}\right)\log 3$

$= \frac{4-1}{8}\log 3$ 1

$= \frac{3}{8}\log 3$ Ans

28 $C(h, 0)$ [∵ Centre lies on x-axis]

Eq^{n} of circle

$(x-h)^2 + y^2 = r^2$ — 1

$2(x-h) + 2y\frac{dy}{dx} = 0$. (diff both sides w.r.t x)

$(x-h) + y\frac{dy}{dx} = 0$ — 1 — 3

$1 + y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

which is the Req. diff eq^{n}. — 1

29 / 30(B)

X : No. of sixes in three throws of a die

$X = 0, 1, 2, 3$

S : Six comes up, $P(S) = \frac{1}{6}$

N : Six does not comes up, $P(N) = \frac{5}{6}$

$P(X=0) = P(NNN)$

$= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$

$= \frac{125}{216}$ — 1

$P(X=1) = P(SNN) + P(NSN) + P(NNS)$

$= \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$

$= 3 \times \frac{25}{216} = \frac{75}{216}$ — 1/2

$P(X=2) = P(SSN) + P(SNS) + P(NSS)$

$= \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times 3$

$= \frac{15}{216}$ — 1/2

$P(X=3) = P(555) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$

X	0	1	2	3
P(X)	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

Mean $\Sigma P_i x_i = \frac{75}{216} + \frac{30}{216} + \frac{3}{216} = \frac{108}{216} = \frac{1}{2}$ — 1

3

30 29(B)

$\bar{x} = 153.7$ (Sample mean)

$\mu = 146.3$ (Population mean) — 1

$n = 400$ (No. of Sample)

$\sigma = 17.2$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{153.7 - 146.3}{\frac{17.2}{20}} = \frac{370}{43}$$

$= 8.605$ (Approx) — 1

3

$Z_\alpha = 1.65$

$Z > Z_\alpha$

$\therefore$ Null hypothesis rejected

$\therefore$ Alternative is accepted — 1

Yes Charge is significant.

31 31(B)

(i) $r = 12$

$k = 2$

$$r_e = \left(1 + \frac{r}{100k}\right)^k - 1$$

$$= \left(1 + \frac{12}{200}\right)^2 - 1 = (1.06)^2 - 1$$

$= 1.1236 - 1 = 0.1236$ — 1

$\therefore$ 12.36% effective rate of return

31

(ii) $r_e = \left(1 + \frac{12}{400}\right)^4 - 1$

$= \left(\frac{412}{400}\right)^4 - 1$

$= (1.03)^4 - 1$

$= 1.125 - 1 = 0.125$ 1

12.5% Rate of return

(iii) $r_e = \left(1 + \frac{12}{1200}\right)^{12} - 1$ 3

$= \left(1 + \frac{1}{100}\right)^{12} - 1$

$= (1.01)^{12} - 1$ 1

$= 1.1268 - 1 = 0.1268$

∴ 12.68% is rate of return

Diff Question of Section C of Set B

26 Equation of ellipse

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

diff both sides w.r.t x

$\frac{2x}{b^2} + \frac{2y}{a^2}\frac{dy}{dx} = 0$ 1

$\frac{x}{b^2} = -\frac{y}{a^2}\frac{dy}{dx}$

$-\frac{a^2}{b^2} = \frac{y}{x}\frac{dy}{dx}$ 1

diff again both sides w.r.t x

$$0 = \frac{y}{x}\frac{d^2y}{dx^2} + \frac{dy}{dx}\left[\frac{x\frac{dy}{dx} - y\times 1}{x^2}\right]$$

$$\frac{y}{x}\frac{d^2y}{dx^2} + \frac{1}{x}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x^2}\frac{dy}{dx} = 0$$

which is the Req. diff eq^n.

Section D

32 $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

$f'(x) = 12x^3 - 12x^2 - 24x$

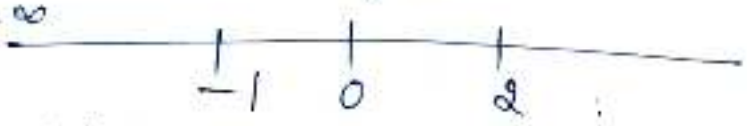
$= 12x[x^2 - x - 2]$

$= 12x[x-2][x+1]$

for C.V.

$f'(x) = 0$

$x = 0, -1, 2$ — 1



$x \in (-\infty, -1)$

$f'(x)$ is $(-ve)(-ve)(-ve) = -ve$

$\therefore f(x)$ is ↓ in $(-\infty, -1)$ — 1

$x \in (-1, 0)$

$f'(x)$ is $(-ve)(-ve)(+ve) = +ve$ — 1

$\therefore f(x)$ is ↑ in $(-1, 0)$

$x \in (0, 2)$

$f'(x)$ is $(+ve)(-ve)(+ve) = -ve$ — 1

$\therefore f(x)$ is ↓ in $(0, 2)$

$x \in (2, \infty)$

$f'(x)$ is $+ve$ $\therefore f(x)$ is ↑ in $(2, \infty)$ — 1

for ↑ $(-\infty, -1) \cup (0, 2)$
for ↓ $(-1, 0) \cup (2, \infty)$

5

or

2(b)

$f(x) = 2x^3 - 21x^2 + 36x - 20$

$f'(x) = 6x^2 - 42x + 36$

$= 6(x^2 - 7x + 6)$

$= 6(x^2 - 6x - x + 6)$

$= 6(x-6)(x-1)$

for c.v

$f'(x) = 0$ 1

$x = 1, 6$

$f''(x) = 12x - 42$

$f''(1) = 12 - 42 = -30 < 0$

$\therefore x = 1$ is the point of local maxima 1 5

L. Maximum value $= 2 - 21 + 36 - 20$

$= 38 - 41$ 1

$= -3$

$f''(6) = 72 - 42 = 30 > 0$

$\therefore x = 6$ is the point of local minima. 1

Local minimum value =

$2(216) - 21(36) + 216 - 20$

$512 - 756 + 216 - 20$ 1

$728 - 776 = -48$

33 $P(X=0)+P(X=1)+P(X=2)+P(X=3)+$ ……

(i) …… $=1$

$k(1)^2+k(2)^2+k(3)^2+2k(4)+2k(5)$

$+2k(6)+0=0$

$k+4k+9k+8k+10k+12k=0$ $1\frac{1}{2}+\frac{1}{2}$

$44k=1$

$k=\frac{1}{44}$

(ii) $P(X<4)$

$P(X=0)+P(X=1)+P(X=2)+P(X=3)$

$0+k+4k+9k$ $1\frac{1}{2}$

$14k=14\times\frac{1}{44}=\frac{7}{22}$

(iii) $P(X\geq 4)$

$P(X=4)+P(X=5)+P(X=6)+P(X=7)$

$+$ ……

$8k+10k+12k+0$ $1\frac{1}{2}$

$30k=30\times\frac{1}{44}=\frac{15}{22}$

34	Part of cistern filled by pipe A in 1 h = $\frac{1}{10}$	1	
	Part of cistern filled by pipe B in 1 h = $\frac{1}{12}$	1	
	Part of cistern emptied by pipe C in 1 h = $\frac{1}{30}$	1	
	Portion of cistern filled by all the pipes A, B & C if all the pipes opened together		5
	$\frac{1}{10} + \frac{1}{12} - \frac{1}{30}$		
	$\frac{6 + 5 - 2}{60} = \frac{9}{60}$	1	
	∴ cistern will be filled in $\frac{60}{9}$ h		
	i.e 6 h and 40 minutes.	1	
35	Actual Price of Car = Downpayment + Balance = 150000 + P (let)	1/2	

Using Reducing Balance method

$$E = \frac{Pi}{1-(1+i)^{-n}}$$ 1

$E = ₹25448$

$i = \frac{28}{5} \times \frac{1}{12} = \frac{1}{60}$

$n = 24$ months

$$25448 = \frac{P \times \frac{20}{120}}{1-\left(1+\frac{20}{120}\right)^{-24}}$$ 1½

$$25448 = \frac{P \times \frac{1}{60}}{1-\left(\frac{61}{60}\right)^{-24}}$$ 5

$$25448 = \frac{\frac{P}{60}}{1-0.67252}$$ 1

$$25448 \times 0.32748 = \frac{P}{60}$$

$P = ₹\ 500023$ (App.)

Total price of Car = 500023 + 150000 = ₹650023 1

36

Section E

No. of Children = x

Amount to be donated to each Child = £y

A.T.Q

$(x-8)(y+10) = xy$

$(x+16)(y-10) = xy$

$xy - 8y + 10x - 80 = xy$

(i) $10x - 8y = 80$

$xy + 16y - 10x - 160 = xy$ $1\frac{1}{2}$

$16y - 10x = 160$

$10x - 16y = -160$

(ii) $AX = B$

$$\begin{bmatrix} 10 & -8 \\ 10 & -16 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 80 \\ -160 \end{bmatrix}$$ 1

(iii) $|A| = -160 + 80 = -80$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$= -\frac{1}{80} \begin{bmatrix} -16 & -10 \\ 8 & 10 \end{bmatrix}$ $1\frac{1}{2}$

$= -\frac{1}{80} \begin{bmatrix} -16 & 8 \\ -10 & 10 \end{bmatrix}$

or

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{80}\begin{bmatrix} -16 & +8 \\ -10 & 10 \end{bmatrix}\begin{bmatrix} 80 \\ -160 \end{bmatrix}$$

$$= -\frac{1}{80}\begin{bmatrix} -1280 + 1280 \\ -800 - 1600 \end{bmatrix}$$

$$= -\frac{1}{80}\begin{bmatrix} -2560 \\ -2400 \end{bmatrix}$$

$$= \begin{bmatrix} 32 \\ 30 \end{bmatrix}$$

$x = 32, \quad y = 30$

1½ | 4

37 (i) $x \geq 0, y \geq 0$ — 1

(ii) (a) $2x - y \leq 0$ and $2x + y \geq 200$ — 2

(b) $x + 2y \leq 100$ and $2x + y \geq 200$

4

(iii) Maximum value of the objective function is 450 — 1

38

(t) Year	y	$x = t - 1998.5$	x^2	xy
1996	6.5	-2.5	6.25	-16.25
1997	5.3	-1.5	2.25	-7.95
1998	4.3	-0.5	0.25	-2.15
1999	6.1	0.5	0.25	3.05
2000	5.6	1.5	2.25	8.4
2001	7.8	2.5	6.25	19.5
	$\Sigma y = 35.6$	$\Sigma x = 0$	$\Sigma x^2 = 17.5$	$\Sigma xy = 4.6$

Middle year $= \frac{1998 + 1999}{2} = 1998.5$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{35.6}{6} = 5.94$$

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{4.6}{17.5} = 0.263$$

The Req. eqn of trend line 2

(i) $y_t = 5.94 + 0.263x$

(ii) 4

(a) Year	1996	1997	1998	1999	2000	2001
Trend Value	5.28	5.54	5.80	6.07	6.33	6.59

For 2002, x shall be 3.5

$$y_{2002} = 5.94 + 0.263(3.5)$$

$$= 5.94 + 0.9205 = 6.86$$

2

Or

(b) Here $n = 7$ (odd)

Assumed mean be 2007

Year (t)	y	x	x^2	xy	$y_t = a + bx$
2004	114	-3	9	-342	115.93
2005	130	-2	4	-260	123.57
2006	126	-1	1	-126	131.21
2007	144	0	0	0	138.85
2008	138	1	1	138	146.49
2009	156	2	4	312	154.13
2010	164	3	9	492	161.77
	972	0	28	214	

$$a = \frac{\Sigma x}{n} = \frac{972}{7} = 138.85$$

$$b = \frac{\Sigma xy}{x^2} = \frac{214}{28} = 7.64$$

The required trend line

$$y_t = 138.85 + 7.64x$$

2