O.S.D.A.V. Public School, Kaithal.
December Exam. 2024-2025
Class : XII
Subject: Mathematics (Core) **Set-A**

Time 3 hrs. **M.M. 80**

General Instructions:

1. This question paper contains **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions
2. **Section A** has **18 MCQ's** and **02 Assertion-Reason based** questions of 1 mark each.
3. **Section B** has **5 Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C has 6 Short Answer (SA)-type** questions of 3 marks each.
5. **Section D has 4 Long Answer (LA)-type** questions of 5 marks each.
6. **Section E has 3 source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION A

1. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is:

   a. 3   b 0   c. -1   d. 1

2. If the matrix A = $\begin{bmatrix} 0 & a & 4 \\ 2 & d & c \\ b & -7 & 0 \end{bmatrix}$ is a skew symmetric matrix, then the value of a + b + c + d is:

   a. 1   b. 2   c. 3   d. 4

3. The total number of possible matrices of order 2 x 3 with each entry 3 or 1 is:

   a. 32   b. 64   c. 512   d. none of these

4. How many reflexive relations are possible in a set A whose n(A) = 3?

   a. 2^3 b.2^6 c. 2^{12} d. 3 !

5. Let R be the equivalence relation in the set A = {0, 1, 2, 3, 4, 5} given by R = {(a, b) : 2 divides (a – b)}. Then the equivalence class of {0} is:

   a. {0, 2, 4}   b. {0, 3, 5}   c. {1, 3, 5}   d. {0, 1, 5}

6. $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}$ dx is:

   a. $\frac{\pi}{12}$ b. $\frac{\pi}{24}$ c. $\frac{\pi}{2}$ d. None of these

7. If the rate of change of volume of a sphere is equal to rate of change of its radius, then its radius is:

a. $\frac{1}{\sqrt{\pi}}$ units b. $\frac{2}{\sqrt{\pi}}$ units c. $\frac{1}{2\sqrt{\pi}}$ units d. 1 unit

8. If p and q are the order and degree of the differential equation :

$$y\frac{dy}{dx} + x^3\frac{d^2y}{dx^2} + xy = \cos x, \text{ then}$$

a. p < q b. p = q c. p > q d. none of these

9. The point which does not lie in the half plane $2x + 3y - 12 \leq 0$ is:

a. (1, 2) b. (2, 1) c. (2, 3) d. (-3, 2)

10. The value of $\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$ is equal to:

a. 0 b. 1 c. -1 d. non of these

11. Two dice are thrown simultaneously. If it is known that sum of numbers on the dice is less than 6, then the probability of getting sum 3 is:

a. $\frac{1}{5}$ b. $\frac{2}{5}$ c. $\frac{5}{18}$ d. $\frac{3}{5}$

12. Find the maximum and minimum value if any, of the function f(x) = - |x+1|+3.

a. Maximum = 4 ; minimum = -1 b. Maximum = 3 ; minimum = -1

c. Maximum = 4 ; minimum = -2 d. Maximum = 3 ; minimum does not exist

13. The number of points where function f(x) = |x + 2| + | x – 3\ is not differentiable is:

a. 2 b. 3 c. 0 d. none of these

14. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined to x-axis at angles of 30° and 120° respectively, then $|\vec{a} + \vec{b}|$ is equal to:

a. 2 b. $\sqrt{2}$ c. $\sqrt{3}$ d. 1

15. If $(\hat{\imath} + \lambda\hat{\jmath})$ x $(5\hat{\imath} + 3\hat{\jmath} + \sigma\hat{k}) = 0$, what are the values λ of and σ?

a. $\lambda = \frac{3}{5}$, $\sigma = 0$ b. $\lambda = \frac{5}{3}$, $\sigma = 5$ c. $\lambda = 3$, $\sigma = 0$

16. Five fair coins are tossed simultaneously. The probability of the events that at least one head comes up is:

a. $\frac{27}{32}$ b. $\frac{5}{32}$ c. $\frac{31}{32}$ d. $\frac{1}{32}$

17. The x-coordinate of a point on the lie joining the points A (2, 2, 1) and B (5, 1, -2) is 4. Find its z-coordinate.

a. 2 b. -1 c. 3 d. 4

18. Let $\vec{a} = x\,\hat{\imath} + 2\,\hat{\jmath} - z\,\hat{k}$ and $\vec{b} = 3\hat{\imath} - y\,\hat{\jmath} + \hat{k}$ be two equal vectors. then x + y + z is equal to:

a. 0 b. 2 c. 3 d. 4

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion: The function f(x) = $\frac{|x|}{x}$ is continous at x = 0.

Reason: The left hand limit and right hand limit of the function f(x) = $\frac{|x|}{x}$ are not equal at x = 0.

20. Assertion: If the events A and b are mutually exclusive such that P(A) = 0.4, P (A ∪ B) = 0.6 and P(B) = p, then p = 0.2

Reasoning: Two events A and B are mutually exclusive if P (A ∪ B) = P(A). P(B).

Section B

21. Show that f : N → N, given by f(x) = $\begin{cases} x+1, & if\ \ x\ is\ odd \\ x-1, & if\ \ x\ is\ even \end{cases}$, is both one-one and onto.

22. Evaluate : $\int \sin 4x \sin 8x \, dx$

23. Find $|\vec{a}|$ and $|\vec{b}|$, if $|\vec{a}| = 2|\vec{b}|$ and $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 12$.

or

If the sum of two unit vectors is a unit vector, prove that the magnitude of their differences is $\sqrt{3}$.

24. Find the equation of the line passing through the point (1, 2, -4) and perpendicular to the two lines:

$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

25. The probabilities of two students A and B coming to school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events ‘A coming in time’ and ‘B coming in time’ are independent, find the probability of only one of them coming to school in time.

or

If E and F are independent events, prove that $\overline{E}$ and $\overline{F}$ are also independent.

Section C

26. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = 0$.

27. Solve the following linear programming problem graphically:
Maximize $Z = 70x + 40y$; subject to constraints $3x + 2y \leq 9$, $3x + y \leq 9$, $x \geq 0$, $y \geq 0$.

28. Find the foot of perpendicular from the point (0, 2, 7) on the line $\frac{x+1}{-1} = \frac{y-1}{3} = \frac{z-3}{-2}$.
Also find the length of the perpendicular.

29. Show that the function $g(x) = |x-2|$, $x \in R$, is continuous but not differentiable at $x = 2$.

or

Let $f(x) = \begin{cases} x^2 + ax + b & 0 \leq x < 2 \\ 3x + 2, & 2 \leq x < 4 \\ 2ax + 5b, & 4 < x \leq 8 \end{cases}$. If f is continuous on [0, 8], find the values of a and b.

30. Evaluate: $\int \frac{x^2}{(x^2+4)(x^2+9)} dx$.

or

Find : $\int \frac{dx}{\sin x + \sin 2x}$.

31. Out of a group of 8 highly qualified doctors in a hospital, 6 are very kind and cooperative with their patients and so are very popular, while the other two remain reserved. For a health camp, three doctors are selected at random. Find the probability distribution of the number of very popular doctors.

Section D

32. Find the intervals in which the function $f(x) = \frac{4 \sin x - 2x - x \cos x}{2 + \cos x}$ is
i. increasing ii. decreasing, where $0 \leq x \leq 2\pi$.

33. Determine whether or not the following pair of lines intersect. If these intersect then find the point of intersection, otherwise obtain the shortest distance between them.
$\vec{r} = (\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda (3\hat{\imath} - \hat{\jmath})$; $\vec{r} = (4\hat{\imath} - \hat{k}) + \mu (2\hat{\imath} + 3\hat{k})$

or

Find the vector equation of a line passing through the point (2, 3, 2) and parallel to the line $\vec{r} = (-2\hat{\imath} + 3\hat{\jmath}) + \lambda(2\hat{\imath} - 3\hat{\jmath} + 6\hat{k})$. Also, find the distance between these lines.

34. Find the area bounded by the curve y = x|x|, x-axis and the ordinates x = -1 and x = 1

or

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$

35. Using properties of definite integral, evaluate $\int_0^{\pi/2}(2\log\sin x - \log\sin 2x)\,dx$

Section E

36. Read the following passage and answer the questions given below:

A coach is training 3 players. He observes that the player A an hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots.

i. Find the probability that all of them will hit the target.

ii. Fid the probability that B and C will hit and A will not hit the target.

iii. What is the probability that any two of A, B and C will hit the target.

or

What is probability that a least one of A, B and C will hit the target?

37. Two schools A and B decided to award prizes to their students for three games-hokey (x), cricket (y) and tennis (z). School A decided to award a total of Rs. 11000 for the three games to 5, 4 and 3 students respectively while school B decided to award Rs. 10700 for the three games to 4, 3 and 5 students respectively. Also the total amount of award for one prize of each game is Rs. 2700. Using the information given above, answer the following:

i. Represent the given situation by a matrix equation.

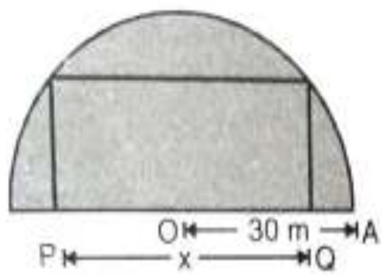
ii. Is the system of equations that represents the given situation consistent or inconsistent.

iv. What is the prize amount for hockey, cricket & Tennis.

or

What will be the total prize amount if there are 2 students each from hockey and criket and only one student from tennis?

38. A gardener plans to plant flowers in a rectangular flower bed in such a way that a rectangle is inscribed in the semi-circular field (as shown in figure). Radius of the semi-circular field is 30m i.e., OA=30m. Let the length of rectangle PQ be ‘x’ m.



Based on above information answer the following:

i. Find the value of x for which the area of the rectangular flower bed is maximum.

ii. Find the area of remaining field (in sq. m) after having the flower bed of maximum area.

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December Exam. 2024-2025

Class : XII

Subject: Mathematics (Core) **Set-B**

Time 3 hrs. **M.M. 80**

General Instructions:

1. This question paper contains **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions
2. **Section A** has **18 MCQ's** and **02 Assertion-Reason based** questions of 1 mark each.
3. **Section B** has **5 Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C has 6 Short Answer (SA)-type** questions of 3 marks each.
5. **Section D has 4 Long Answer (LA)-type** questions of 5 marks each.
6. **Section E has 3 source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION A

1. If A = $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, B = $\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and A = B^2 then x equals:

   a. ± 1 b. -1 c. 1 d. 2

2. Find the maximum and minimum value if any, of the function f(x) = - |x+1|+3.

   a. Maximum = 4 ; minimum = -1 b. Maximum = 3 ; minimum = -1

   c. Maximum = 4 ; minimum = -2 d. Maximum = 3 ; minimum does not exist

3. If the rate of change of volume of a sphere is equal to rate of change of its radius, then its radius is:

   a. $\frac{1}{\sqrt{\pi}}$ units b. $\frac{2}{\sqrt{\pi}}$ units c. $\frac{1}{2\sqrt{\pi}}$ units d. 1 unit

4. If the matrix A = $\begin{bmatrix} 0 & a & 4 \\ 2 & d & c \\ b & -7 & 0 \end{bmatrix}$ is a skew symmetric matrix, then the value of a + b + c + d is:

   a. 1 b. 2 c. 3 d. 4

5. How many reflexive relations are possible in a set A whose n(A) = 3?

   a. 2^3 b. 2^6 c. 2^{12} d. 3 !

6. $\int e^{\frac{1}{2}\log x}$ dx is equal to:

a. $x^2 + c$ b. $\frac{2}{3}x^{3/2} + c$ c. $\frac{x^2}{2} + c$ d. $2\sqrt{x}+c$

7. The total number of possible matrices of order 2 x 3 with each entry 3 or 1 is:

a. 32 b. 64 c. 512 d. none of these

8. The product of order and degree of differential equation $y = \frac{dy}{dx} + \frac{a}{\frac{dy}{dx}}$ is:

a. 1 b. 2 c. 3 d. 4

9. The value of $\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$ is equal to:

a. 0 b. 1 c. -1 d. none of these

10. Two dice are thrown simultaneously. If it is known that sum of numbers on the dice is less than 6, then the probability of getting sum 3 is:

a. $\frac{1}{5}$ b. $\frac{2}{5}$ c. $\frac{5}{18}$ d. $\frac{3}{5}$

11. The point which does not lie in the half plane $2x + 3y - 12 \leq 0$ is:

a. (1, 2) b. (2, 1) c. (2, 3) d. (-3, 2)

12. The number of points where function f(x) = |x + 2| + | x – 3\ is not differentiable is:

a. 2 b. 3 c. 0 d. none of these

13. If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, then the value of $|\vec{a} \cdot \vec{b}|$ is: is equal to:

a. 2 b. 4 c. 6 d. 8

14. Let R be the equivalence relation in the set A = {0, 1, 2, 3, 4, 5} given by R = {(a, b) : 2 divides (a – b)}. Then the equivalence class of {0} is:

a. {0, 2, 4} b. {0, 3, 5} c. {1, 3, 5} d. {0, 1, 5}

15. Five fair coins are tossed simultaneously. The probability of the events that at least one head comes up is:

a. $\frac{27}{32}$ b. $\frac{5}{32}$ c. $\frac{31}{32}$ d. $\frac{1}{32}$

16. Let $\vec{a} = x\,\hat{\imath} + 2\,\hat{\jmath} - z\,\hat{k}$ and $\vec{b} = 3\hat{\imath} - y\,\hat{\jmath} + \hat{k}$ be two equal vectors. then x + y + z is equal to:

a. 0 b. 2 c. 3 d. 4

17. If $(\hat{\imath} + \lambda\hat{\jmath}) \times (5\,\hat{\imath} + 3\hat{\jmath} + \sigma\hat{k}) = 0$, what are the values λ of and σ?

a. $\lambda = \frac{3}{5}$, $\sigma = 0$ b. $\lambda = \frac{5}{3}$, $\sigma = 5$ c. $\lambda = 3, \sigma = 0$

18. The x-coordinate of a point on the lie joining the points A (2, 2, 1) and B (5, 1, -2) is 4. Find its z-coordinate.

a. 2   b. -1   c. 3   d. 4

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion: If the events A and b are mutually exclusive such that

P(A) = 0.4, P (A ∪ B) = 0.6 and P(B) = p, then p = 0.2

Reasoning: Two events A and B are mutually exclusive if P (A ∪ B) = P(A) . P(B).

20. Assertion: The function f(x) = $\frac{|x|}{x}$ is continous at x = 0.

Reason: The left hand limit and right hand limit of the function f(x) = $\frac{|x|}{x}$ are not equal at x = o.

Section B

21. Find | $\vec{a}$ | and | $\vec{b}$ |, if | $\vec{a}$ | = 2 | $\vec{b}$ | and ($\vec{a}$ + $\vec{b}$) . ($\vec{a}$ - $\vec{b}$) = 12.

or

If the sum of two unit vectors is a unit vector, prove that the magnitude of their differences is $\sqrt{3}$.

22. Show that f : N → N, given by f(x) = $\begin{cases} x+1, & if\ \ x\ is\ odd \\ x-1, & if\ \ x\ is\ even \end{cases}$, is both one-one and onto.

23. Evaluate : $\int \frac{x^3 \sin(tan^{-1}x^4}{1+x^8}$ dx

24. The probabilities of two students A and B coming to school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events 'A coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to school in time.

or

If E and F are independent events, prove that $\overline{E}$ and $\overline{F}$ are also independent.

25. Find the vector equation of the line passing through the point A (1, 2, -1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.

Section C

26. Evaluate: $\int \frac{3x+1}{(x+3)(x-1)^2}\, dx$.

or

Find : $\int \frac{dx}{\sqrt{\sin^3 x \sin(x+a)}}$.

27. Solve the following linear programming problem graphically:

Maximize $= 150x + 250y$

Subject to the constraints $x + y \le 35$, $x + 2y \le 50$, $x \ge 0$, $x \ge 0$

28. Find the foot of perpendicular from the point (0, 2, 7) on the line $\frac{x+1}{-1} = \frac{y-1}{3} = \frac{z-3}{-2}$.

Also find the length of the perpendicular.

29. Out of a group of 8 highly qualified doctors in a hospital, 6 are very kind and cooperative with their patients and so are very popular, while the other two remain reserved. For a health camp, three doctors are selected at random. Find the probability distribution of the number of very popular doctors.

30. Show that the function $g(x)=|x-2|$, $x\in R$, is continuous but not differentiable at $x = 2$.

or

Let $f(x) = \begin{cases} x^2 + ax + b & 0 \le x < 2 \\ 3x + 2, & 2 \le x < 4 \\ 2ax + 5b, & 4 < x \le 8 \end{cases}$. If f is continuous on [0, 8], find the values of a and b.

31. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = 0$.

Section D

32. Find the equations of the diagonals of the parallelogram PQRS whose vertices are P(4, 2, -6), Q (5, -3, 1), R (12, 4, 5) and S (11, 9, -2). Use these equations to find the point of intersection of diagonals.

33. Find the intervals in which the function f(x) = $\frac{4\sin x-2x-x\cos x}{2+\cos x}$ is

i. increasing ii. decreasing, where $0 \le x \le 2\pi$.

34. Using properties of definite integral, evaluate $\int_0^{\pi/2}(2\log\sin x-\log\sin 2x)\,dx$

35. Find the area bounded by the curve y = x|x|, x-axis and the ordinates x = -1 and x = 1

or

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$

Section E

36. Rakesh is playing the game of snooker. On the snooker table, there are 12 blue balls, 8 red balls, 10 yellow balls and 5 green balls. If he picks two balls, one after the other, without replacement, then answer the following questions:

i. Find the probability that the first ball is blue and the second ball is green.

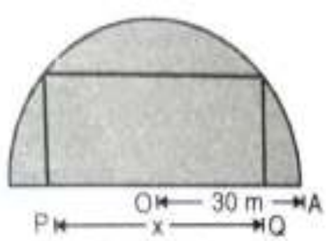
ii. What is the probability that the first ball is yellow and the second ball is red?

iii. Find the probability that the first ball is green and the second ball is not yellow.

or

What is the probability that both the balls are not blue?

37. A gardener plans to plant flowers in a rectangular flower bed in such a way that a rectangle is inscribed in the semi-circular field (as shown in figure). Radius of the semi-circular field is 30m i.e., OA=30m. Let the length of rectangle PQ be 'x' m.



Based on above information answer the following:

i. Find the value of x for which the area of the rectangular flower bed is maximum.

ii. Find the area of remaining field (in sq. m) after having the flower bed of maximum area.

38. Two schools A and B decided to award prizes to their students for three games-hokey (x), cricket (y) and tennis (z). School A decided to award a total of Rs. 11000 for the three games to 5, 4 and 3 students respectively while school B decided to award Rs. 10700 for the three games to 4, 3 and 5 students respectively. Also the total amount of award for one prize of each game is Rs. 2700. Using the information given above, answer the following:

i. Represent the given situation by a matrix equation.

ii. Is the system of equations that represents the given situation consistent or inconsistent.

i. What is the prize amount for hockey.

or

What will be the total prize amount if there are 2 students each from hockey and criket and only one student from tennis?

December Exam: 2024-25

Mathematics (Core) Set A and B

Marking Scheme / Hints to Solutions

Note: Any other relevant answer not given here in but given by the students are suitably awarded.

S.No.	Section A Value points / Key points	Marks allotted to each key point	Total points
1	(c) -1	1	1
2 4(B)	(a) 1	1	1
3 7(B)	(b) 64	1	1
4 5(B)	(b) 2^6	1	1
5 14(B)	(a) $\{0, 2, 4\}$	1	1
6	(a) $\pi/12$	1	1
7 3(B)	(c) $\frac{1}{2\sqrt{\pi}}$ units	1	1
8	(a) $p < q$	1	1
9 11(B)	(c) $(2, 3)$	1	1
10 9(B)	(c) -1	1	1
11	(a) $\frac{1}{5}$	1	1
12 2(B)	(d) Maximum $= 3$; minimum does not exist.	1	1
13 12(B)	(a) 2	1	1

14	(b) $\sqrt{2}$	1	1
15 11(B)	(a) $d=\frac{3}{5}, c=0$	1	1
16 15(B)	(c) $\frac{31}{32}$	1	1
17 18(B)	(b) -1	1	1
18 16(B)	(a) 0	1	1
19 20(B)	(d) A is false but R is true	1	1
20 19(B)	(c) A is True but R is false	1	1
	Section A Different Questions of Set B		
1	(c) 1	1	1
6	(b) $\frac{2}{3}x^{3/2}+C$	1	1
8	(b) 2	1	1
13	(c) 6	1	1

21
22(B)

Section B

$f: N \to N \quad f(x) = \begin{cases} x+1 & \text{if } x \text{ is odd} \\ x-1 & \text{if } x \text{ is even} \end{cases}$

Let $x_1, x_2 \in N$ (Domain)

Case(i) x_1 & x_2 both are odd

s.t. $f(x_1) = f(x_2)$

$x_1 + \cancel{1} = x_2 + \cancel{1}$

$x_1 = x_2$

$\therefore f$ is one-one

Case(i) $x_1, x_2 \in$ even No.

s.t $f(x_1) = f(x_2)$

$x_1 - \cancel{1} = x_2 - \cancel{1}$

$x_1 = x_2$

$\therefore f$ is one-one 1/2

Case(ii) x_1 is even & x_2 is odd

$\therefore x_1 \neq x_2$

$f(x_1) = x_1 - 1$

$f(x_2) = x_2 + 1$ $\therefore f(x_1) \neq f(x_2)$

$\therefore f$ is one-one

Case(iii) x_1 is odd & x_2 is even

$\therefore x_1 \neq x_2$

$f(x_1) = x_1 + 1$, $f(x_2) = x_2 - 1$

$\therefore f(x_1) \neq f(x_2)$ 1/2 2

$\therefore f$ is one-one

$\forall$ $y \in N$ (Co-domain)

s.t $f(x) = y$

$x + 1 = y$ if x is odd

$x = y - 1 \in N$ (domain)

$y = x - 1$ if x is even

$x = y + 1 \in N$ (domain) 1

$\therefore f$ is onto

22	$\frac{1}{2}\int 2\sin 4x \sin 8x\,dx$	1/2	2
	$\frac{1}{2}\int \cos(4x) - \cos(12x)\,dx$	1/2	
	$\frac{1}{2}\left[\frac{\sin 4x}{4} - \frac{\sin 12x}{12}\right] + C$	1	
23 21(B)	$\lvert\vec{a}\rvert = 2\lvert\vec{b}\rvert$		2
	$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 12$		
	$\vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} - \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b} = 12$	1/2	
	$\lvert\vec{a}\rvert^2 + \cancel{\vec{a}\cdot\vec{b}} - \cancel{\vec{a}\cdot\vec{b}} - \lvert\vec{b}\rvert^2 = 12$ [$\because$ dot product is Commutative]	1/2	
	$[2\lvert\vec{b}\rvert]^2 - \lvert\vec{b}\rvert^2 = 12$	1/2	
	$3\lvert\vec{b}\rvert^2 = 12 \Rightarrow \lvert\vec{b}\rvert^2 = 4$, $\lvert\vec{b}\rvert = 2$	1/2	
	$\therefore \lvert\vec{a}\rvert = 4$		
	or		
	$\lvert\vec{a}\rvert = 1$ & $\lvert\vec{b}\rvert = 1$ & $\lvert\vec{a}+\vec{b}\rvert = 1$ (A.T.Q)		
	T.P. $\lvert\vec{a}-\vec{b}\rvert = \sqrt{3}$	1/2	
	$\lvert\vec{a}+\vec{b}\rvert^2 = 1$		
	$(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = 1$		
	$\vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b} = 1$		
	$\lvert\vec{a}\rvert^2 + 2\vec{a}\cdot\vec{b} + \lvert\vec{b}\rvert^2 = 1$ [dot product is Commutative]		
	$\cancel{1} + 1 + 2\vec{a}\cdot\vec{b} = \cancel{1}$	1/2	
	$\vec{a}\cdot\vec{b} = -1/2$		

	$\|\vec{a}-\vec{b}\|^2 = (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$	1/2	
	$= \vec{a}\cdot\vec{a} - \vec{b}\cdot\vec{a} - \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b}$		
	$= \|\vec{a}\|^2 - 2\vec{a}\cdot\vec{b} + \|\vec{b}\|^2$ [Dot product is Commutative]		
	$= 1 - 2\times\left(\frac{-1}{2}\right) + 1$		2
	$= 3$		
	$\|\vec{a}-\vec{b}\| = \sqrt{3}$	1/2	
24	Equation of Req. line		
	$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c}$	1/2	
	Req. line is $\perp$ to the lines		
	$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$		
	$\therefore 3a - 16b + 7c = 0$		
	$3a + 8b - 5c = 0$	1/2	2
	$\frac{a}{80-56} = \frac{b}{21+15} = \frac{c}{24+48}$		
	$\frac{a}{24} = \frac{b}{36} = \frac{c}{72}$		
	$\langle a, b, c\rangle = \langle 24, 36, 72\rangle$	1/2	
	i.e $\langle 2, 3, 6\rangle$		
	$\therefore$ eqn of Req line		
	$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$	1/2	

25 E: A coming in time

24(B) F: B " " "

$P(E) = \frac{3}{7}$ $P(F) = \frac{5}{7}$

$P(E \cap \bar{F}) + P(\bar{E} \cap F)$ 1/2

$P(E) - P(E \cap F) + P(F) - P(E \cap F)$ 1/2

$P(E) + P(F) - 2P(E \cap F)$

$P(E) + P(F) - 2P(E) \cdot P(F)$ 1/2 2

$\frac{3}{7} + \frac{5}{7} - 2 \times \frac{3}{7} \times \frac{5}{7}$ [∵ E & F are independent]

$\frac{21 + 35 - 30}{49} = \frac{26}{49}$ 1/2

OR

Given:- E & F are independent

∴ $P(E \cap F) = P(E) \cdot P(F)$

T.P.T. $\bar{E}$ & $\bar{F}$ are independent

i.e. $P(\bar{E} \cap \bar{F}) = P(\bar{E}) \cdot P(\bar{F})$ 1/2

$P(\bar{E} \cap \bar{F}) = P(\overline{E \cup F})$

$= 1 - P(E \cup F)$ 1/2

$= 1 - P(E) - P(F) + P(E \cap F)$

$= [1 - P(E)] - P(F) + P(E) \cdot P(F)$ 1/2

$= [1 - P(E)] - P(F)[1 - P(E)]$

	$[1-P(E)][1-P(F)] = P(\bar{E})\cdot P(\bar{F})$	1/2	2
	Section B Different Questions of Set B		
23	$\int \frac{x^3 \sin(\tan^{-1}x^4)}{1+x^8}dx$ put $\tan^{-1}x^4 = t$ $\frac{4x^3}{1+x^8}dx = dt$ $\frac{1}{4}\int \sin t\, dt$ $\qquad$ $\frac{x^3}{1+x^8}dx = \frac{dt}{4}$ $-\frac{1}{4}\cos t + c$ $-\frac{1}{4}\cos(\tan^{-1}x^4) + c$	1 1	2
25	Equation of Req. line $\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+1}{c}$ Req. line is $\parallel$ to the line $5x-25 = 14-7y = 35z$ $5(x-5) = -7(y-2) = 35(z-0)$ $\frac{x-5}{1/5} = \frac{y-2}{-1/7} = \frac{z-0}{1/35}$ $\therefore \langle a, b, c\rangle = \langle \frac{1}{5}, -\frac{1}{7}, \frac{1}{35}\rangle$ i.e $\langle 7, -5, 1\rangle$ $\therefore$ eqn of Req. line $\frac{x-1}{7} = \frac{y-2}{-5} = \frac{z+1}{1}$	1/2 1/2 1/2 1/2	2

Section C

26

31(B)

$$A^2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$ 1

$$A^2 - 5A + 4I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$ 3

$$= \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$ 1

$$\begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix} * X = 0$$

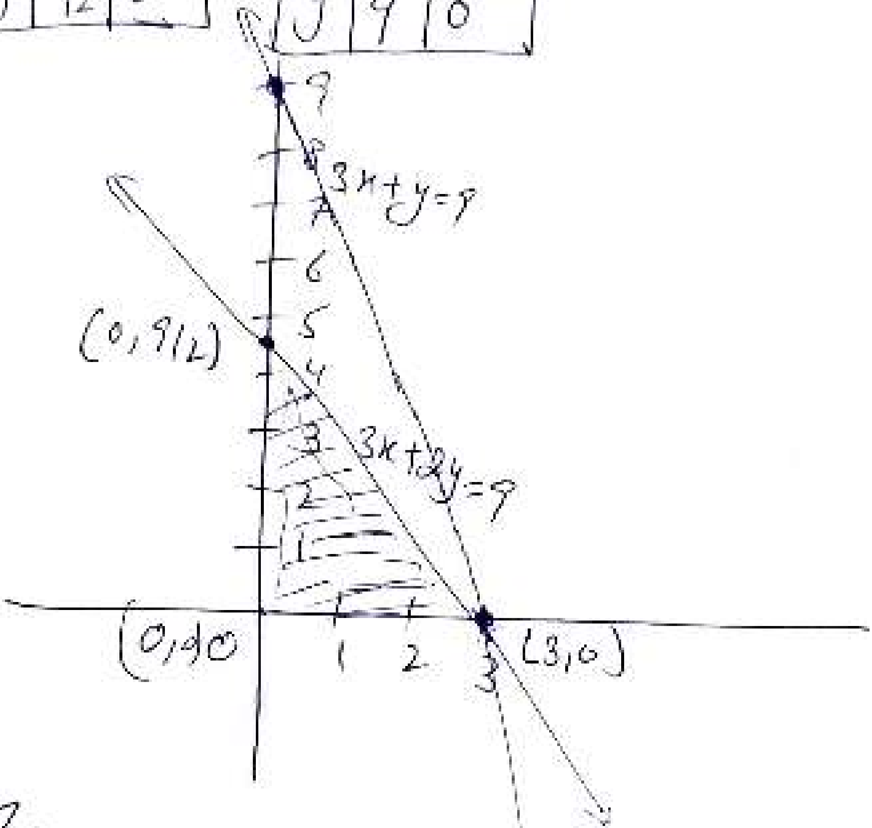
$$X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{bmatrix}$$ 1

27	$Z = 70x + 40y$ (Maximize)	
	$3x+2y \le 9,\ 3x+y \le 9,\ x \geq 0, y \geq 0$	
	$3x+2y=9$	

x	0	3
y	$9/2$	0

x	0	3
y	9	0

1/2



1 1/2

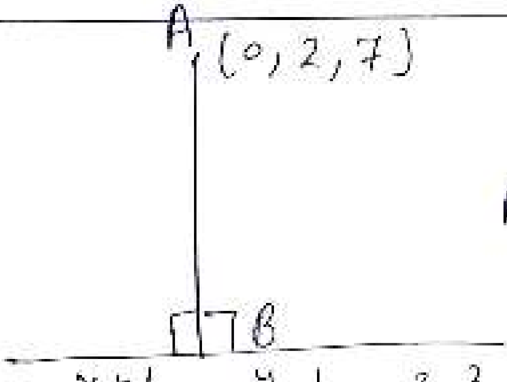
$Z_{(0,0)} = 0 \quad Z_{(3,0)} = 210$ (Maximum)

$Z_{(0,\frac{9}{2})} = 70(0) + \cancel{40}^{20}\left(\frac{9}{2}\right) = 180$

Maximum value 210, $x=3, y=0$

28

28(B)



$\frac{x+1}{-1} = \frac{y-1}{3} = \frac{z-3}{-2}$ — (1)

Any point on line (1)

$B(-k-1, 3k+1, -2k+3)$ 1/2

D. Ratios of AB

$\langle -k-1, 3k+1, -2k-4 \rangle$

1/2

AB is ⊥ (1)

$-1(-k-1)+3(3k-1)-2(-2k-4)=0$

$k+1+9k-3+4k+8=0$

$14k=-6$

$k=-\frac{6}{14}=-\frac{3}{7}$ $\quad 1/2$

$B\left(\frac{3}{7}-1,\ -\frac{9}{7}+1,\ \frac{6}{7}+3\right)$

$B\left(-\frac{4}{7},\ -\frac{2}{7},\ \frac{27}{7}\right)$ $\quad 1/2$

$AB=\sqrt{\left(-\frac{4}{7}-0\right)^2+\left(-\frac{2}{7}-2\right)^2+\left(\frac{27}{7}-7\right)^2}$ $\quad 1/2$ $\quad 3$

$=\sqrt{\frac{16}{49}+\frac{256}{49}+\frac{484}{49}}$

$=\sqrt{\frac{16+256+484}{49}}=\sqrt{\frac{756}{49}}$

$=\frac{\sqrt{756}}{7}$ $\quad 1/2$

29
30(B) $g(x)=|x-2|,\ x\in R$

L.H.L	R.H.L	$g(2)=\lvert 0\rvert$
$\lim_{x\to 2^-}\lvert x-2\rvert$	$\lim_{x\to 2^+}\lvert x-2\rvert$	$=0$
put $x=2-h$	put $x=2+h$	
$\lim_{h\to 0}\lvert 2-h-2\rvert$	$\lim_{h\to 0}\lvert 2+h-2\rvert$	
$\lim_{h\to 0} h$	$\lim_{h\to 0} h$	
0	0	

$1\frac{1}{2}$

$\therefore \lim_{x\to 2} g(x)=g(2)$ $\quad \therefore g(x)$ is cts at $x=2$

	L.H.D $\lim_{x \to 2^-} \frac{f(x)-f(2)}{x-2}$ $\lim_{x \to 2^-} \frac{\|x-2\|-0}{x-2}$ $x = 2-h$ $\lim_{h \to 0} \frac{\|\cancel{2}-h-\cancel{2}\|}{\cancel{2}-h-\cancel{2}}$ $\lim_{h \to 0} \frac{\cancel{h}}{-\cancel{h}} = -1$ R.H.D $\lim_{x \to 2^+} \frac{f(x)-f(2)}{x-2}$ $\lim_{x \to 2^+} \frac{\|x-2\|-0}{x-2}$ $x = 2+h$ $\lim_{h \to 0} \frac{\|\cancel{2}+h-\cancel{2}\|}{\cancel{2}+h-\cancel{2}}$ $\lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}} = 1$ L.H.D $\neq$ R.H.D $\therefore f(x)$ is not diff at $x = 2$	$1\frac{1}{2}$	3
29	Or $f(x) = \begin{cases} x^2+ax+b & 0 \le x < 2 \\ 3x+2 & 2 \le x < 4 \\ 2ax+5b & 4 < x \le 8 \end{cases}$ L.H.L $\lim_{x \to 2^-} x^2+ax+b$ $4+2a+b$ R.H.L $\lim_{x \to 2^+} 3x+2$ 8 $f(x)$ is cts on $[0,8]$ ——— (1) $\therefore 4+2a+b = 8$ $5 \times 2a+b = 4$	1	

L.H.L	R.H.L
$\lim_{x\to 4^-} 3x+2$	$\lim_{x\to 4^+} 2ax+5b$
14	$8a+5b$

1

$$8a+5b=14 \quad \text{---} (1)$$
$$\underline{\_10a+5b=20}$$
$$+2a = +6$$
$$a=3$$
$$24+5b=14$$
$$5b=-10$$
$$b=-2$$

1

$$a=3,\ b=-2$$

3

30 $\int \frac{x^2}{(x^2+4)(x^2+9)} dx$

let $x^2=t$ 1/2

$\frac{x^2}{(x^2+4)\cdot(x^2+9)}$ 1/2

$$\frac{t}{(t+4)(t+9)} = \frac{A}{t+4} + \frac{B}{t+9}$$

$$t = A(t+9) + B(t+4)$$

put $t=-9,-4$

$+9 = +5B$ | $-4 = 5A$

$\frac{9}{5} = B$ | $-\frac{4}{5} = A$

$$-\frac{4}{5}\int \frac{1}{x^2+4} dx + \frac{9}{5}\int \frac{1}{x^2+9} dx$$ 1

$-\frac{4}{5}\times\frac{1}{2}\tan^{-1}\frac{x}{2}+\frac{9}{5}\times\frac{1}{3}\tan^{-1}\frac{x}{3}+c$ | | 3

$-\frac{2}{5}\tan^{-1}\frac{x}{2}+\frac{3}{5}\tan^{-1}\frac{x}{3}+c$ | 1 |

or

30 $\int\frac{dx}{\sin x+\sin 2x}$

$\int\frac{dx}{\sin x+2\sin x\cos x}$

$\int\frac{dx\,(\sin x)}{\sin^2 x(1+2\cos x)}$ | 1/2

$\cos x=t$

$-\sin x\,dx=dt$

$-\int\frac{dt}{(1-t^2)(1+2t)}$ | 1/2

$-\int\frac{dt}{(1-t)(1+t)(1+2t)}$

$\frac{1}{(1-t)(1+t)(1+2t)}=\frac{A}{1-t}+\frac{B}{1+t}+\frac{C}{1+2t}$

$1=A(1+t)(1+2t)$

$+B(1-t)(1+2t)$

$+C(1-t)(1+t)$

$t=1,-1,-\frac{1}{2}$

$1 = A(8) \Rightarrow A = 1/6$

$1 = -2B \Rightarrow B = -1/2$

$1 = \frac{3}{4} C \Rightarrow C = 4/3$

$+\frac{1}{6}\int \frac{-1}{1-t}\,dt + \frac{1}{2}\int \frac{1}{1+t}\,dt - \frac{4}{3}\times\frac{1}{2}\int \frac{2\,dt}{1+2t}$	1	
$\frac{1}{6}\log\lvert 1-t\rvert + \frac{1}{2}\log\lvert 1+t\rvert - \frac{2}{3}\log\lvert 1+2t\rvert + C$		3
$\frac{1}{6}\log\lvert 1-\cos x\rvert + \frac{1}{2}\log\lvert 1+\cos x\rvert - \frac{2}{3}\log\lvert 1+2\cos x\rvert + C$	1	

31 X: No. of Popular doctors

23(B) $X: 1, 2, 3$

$P(X=1) = \dfrac{{}^6C_1 \times {}^2C_2}{{}^8C_3} = \dfrac{6 \times 1}{\dfrac{\lfloor 8}{\lfloor 3 \lfloor 5}}$

$= \dfrac{6 \times \cancel{6}}{8 \times 7 \times \cancel{6}} = \dfrac{\cancel{6}\,3}{\cancel{56}\,28}$

$P(X=2) = \dfrac{{}^6C_2 \times {}^2C_1}{{}^8C_3} = \dfrac{\dfrac{\lfloor 6}{\cancel{\lfloor 2}\lfloor 4} \times \cancel{2}}{\dfrac{\lfloor 8}{\lfloor 3 \lfloor 5}}$

$= \dfrac{6 \times 5}{8 \times 7 \times \cancel{6}} \times \cancel{6} = \dfrac{15}{\cancel{56}\,28}$

$P(X=3) = \dfrac{{}^6C_3 \times {}^2C_0}{{}^8C_3} = \dfrac{\dfrac{\lfloor 6}{\cancel{\lfloor 3}\lfloor 3} \times 1}{\dfrac{\lfloor 8}{\cancel{\lfloor 3} \lfloor 5}}$

$= \dfrac{\cancel{6} \times 5 \times 4}{8 \times 7 \times \cancel{6}} = \dfrac{\cancel{20}\,10}{\cancel{56}\,28}$

X	1	2	3
P(X)	$\frac{3}{28}$	$\frac{15}{28}$	$\frac{10}{28}$

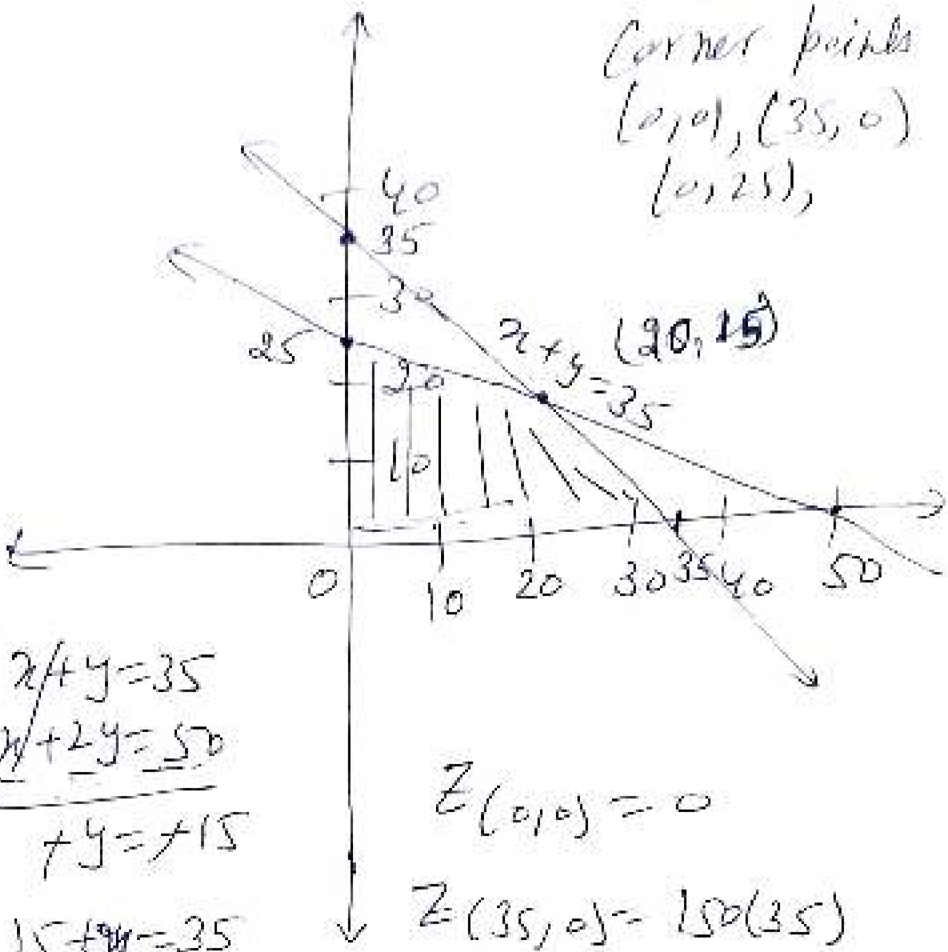
Different questions of Section C of Set B

27 $Z = 150x + 250y$

$x + y \leq 35 \quad x + 2y \leq 50 \quad x, y \geq 0$

x	0	35
y	35	0

x	0	50
y	25	0



Corner points
(0,0), (35,0)
(0,25),

$1\frac{1}{2}$

$x+y=35$
$x+2y=50$
$+y=+15$

$15+x=35$
$x=20$

3

$Z(0,0)=0$

$Z(35,0)=150(35)$
$=5250$

$Z(0,25)=250(25)$ $1\frac{1}{2}$
$=6250$

$Z(20,15)=150(20)+250(15)$
$=3000+3750$
$=6750$

---

26 $\int\frac{3x+1}{(x+3)(x-1)^2}dx$

$\frac{3x+1}{(x+3)(x-1)^2}=\frac{A}{x+3}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2}$ 1

$3x+1=A(x-1)^2+B(x+3)(x-1)+C(x+3)$

$x = 1, -3$

$4 = 4C \Rightarrow C = 1$

$-8 = 16A \Rightarrow A = -1/2$

Compare the co-eff of x

$3 = -2A + 2B + C$

$3 = +\cancel{2}\times\frac{1}{\cancel{2}} + 2B + 1$

$3 - 2 = 2B \Rightarrow B = 1/2$ — 1

$-\frac{1}{2}\int\frac{1}{x+3}dx + \frac{1}{2}\int\frac{1}{x-1}dx + \int\frac{1}{(x-1)^2}dx$ — 3

$-\frac{1}{2}\log|x+3| + \frac{1}{2}\log|x-1| - \frac{1}{x-1} + C$

$\frac{1}{2}\log\left|\frac{x-1}{x+3}\right| - \frac{1}{x-1} + C$ — Ans — 1

---

or

$\int\frac{dx}{\sqrt{\sin^3 x\sin(x+\alpha)}} = \int\frac{dx}{\sqrt{\sin^3 x[\sin x\cos\alpha + \cos x\sin\alpha]}}$ — 1/2

$\int\frac{dx}{\sqrt{\sin^4 x[\cos\alpha + \cot x\sin\alpha]}}$ — 1/2

$\int\frac{\operatorname{cosec}^2 x\,dx}{\sqrt{\cos\alpha + \cot x\sin\alpha}}$ — 3

put $\cos\alpha + \cot x\sin\alpha = t$ — 1

$-\sin\alpha\operatorname{cosec}^2 x\,dx = dt$

$\frac{-1}{\sin\alpha}\int\frac{dt}{\sqrt{t}}$

$\frac{-1}{\sin\alpha}2t^{1/2} + C = \frac{-2}{\sin\alpha}\sqrt{\cos\alpha + \cot x\sin\alpha} + C$ — Ans — 1

Section D

32 $f(x) = \frac{4\sin x - 2x - x\cos x}{2+\cos x}$

33(B) $f(x) = \frac{4\sin x}{2+\cos x} - \frac{x(2+\cos x)}{(2+\cos x)}$

$f'(x) = \frac{(2+\cos x)(4\cos x) - 4\sin x(-\sin x)}{(2+\cos x)^2} - 1$

$= \frac{8\cos x + 4\cos^2 x + 4\sin^2 x}{(2+\cos x)^2} - 1$

$= \frac{8\cos x + 4}{(2+\cos x)^2} - \frac{1}{1}$

$= \frac{8\cos x + \cancel{4} - \cancel{4} - \cos^2 x - 4\cos x}{(2+\cos x)^2}$

$= \frac{4\cos x - \cos^2 x}{(2+\cos x)^2}$

$= \frac{\cos x(4-\cos x)}{(2+\cos x)^2}$ 2

5

$(2+\cos x)^2 \geq 0$ ∵ Square of any no. is +ve 1

$4-\cos x \geq 0$ ∵ $-1 \leq \cos x \leq 1$

$\cos x$ is +ve in $(0, \pi/2) \cup (3\pi/2, 2\pi)$

∴ $f(x)$ is ↑ in $(0, \pi/2) \cup (3\pi/2, 2\pi)$ 1

$\cos x$ is −ve in $(\pi/2, 3\pi/2)$ 1

∴ $f(x)$ is ↓ in $(\pi/2, 3\pi/2)$

33 $\vec{r} = (\hat{i}+\hat{j}-\hat{k}) + \lambda(3\hat{i}-\hat{j})$

$\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda$ — (1)

$\vec{r} = (4\hat{i}-\hat{k}) + \mu(2\hat{i}+3\hat{k})$

$\frac{x-4}{2} = \frac{y-0}{0} = \frac{z+1}{3} = \mu$ — (2)

Any point on line (1)

$(3\lambda+1, -\lambda+1, -1)$ 1/2

Any point of line (2)

$(2\mu+4, 0, 3\mu-1)$ 1/2 5

if lines (1) & (2) intersect

$3\lambda+1 = 2\mu+4$ $\quad -\lambda+1=0$ $\quad 3\mu-1=-1$

$\lambda=1, \mu=0$ $\quad -\lambda=-1$ $\quad 3\mu=0$

$4=4$ $\quad \lambda=1$ $\quad \mu=0$

which is true

$\therefore$ lines are intersect each other at $(4, 0, -1)$ 2

or

33 Passing point of Req line $(2,3,2)$

Req. line is $\parallel$ to the line

$\vec{r} = (-2\hat{i}+3\hat{j}) + \lambda(2\hat{i}-3\hat{j}+6\hat{k})$

$\therefore$ eqn of Req. line

$\vec{r} = (2\hat{i}+3\hat{j}+2\hat{k}) + \mu(2\hat{i}-3\hat{j}+6\hat{k})$ 1

$\vec{a_2} - \vec{a_1} = 4\hat{i} + 0\hat{j} + 2\hat{k}$

$\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$d = \left| \dfrac{(\vec{a_2} - \vec{a_1}) \times \vec{b}}{|\vec{b}|} \right|$ 1½

$(\vec{a_2} - \vec{a_1}) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 2 \\ 2 & -3 & 6 \end{vmatrix}$

$\hat{i}[6] - \hat{j}[20] + \hat{k}[-12]$ 1

$6\hat{i} - 20\hat{j} - 12\hat{k}$

$|\vec{b}| = \sqrt{4+9+36} = 7$ ½

$d = \left| \dfrac{6\hat{i} - 20\hat{j} - 12\hat{k}}{7} \right|$

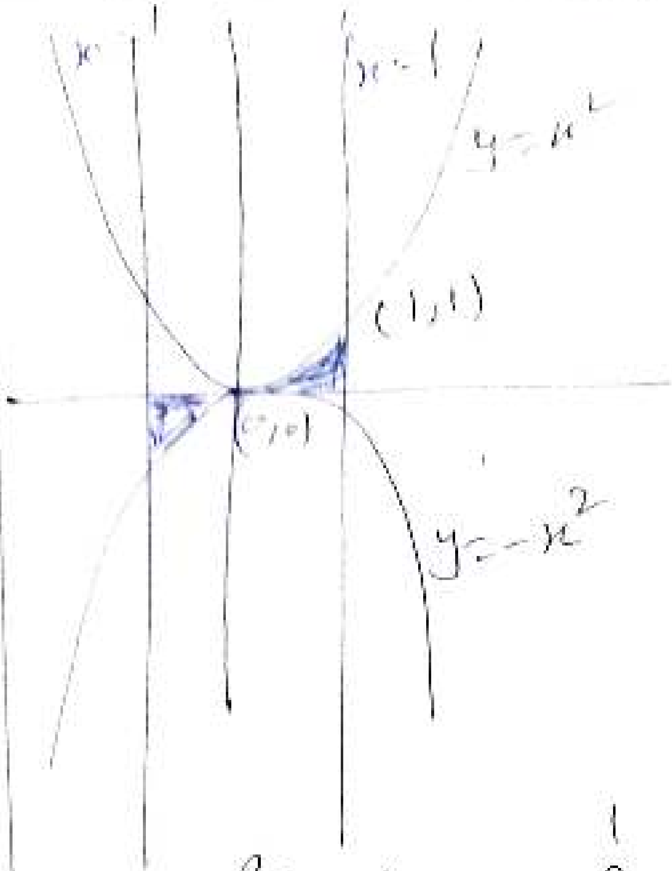
$= \dfrac{\sqrt{36+400+144}}{7}$

$= \dfrac{\sqrt{580}}{7}$ — Ans 1

5

34 / 35(B) $y = x|x|$

$y = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$ 1



2

Req. Area $= 2\int_0^1 x^2\,dx$ | 1 | 5

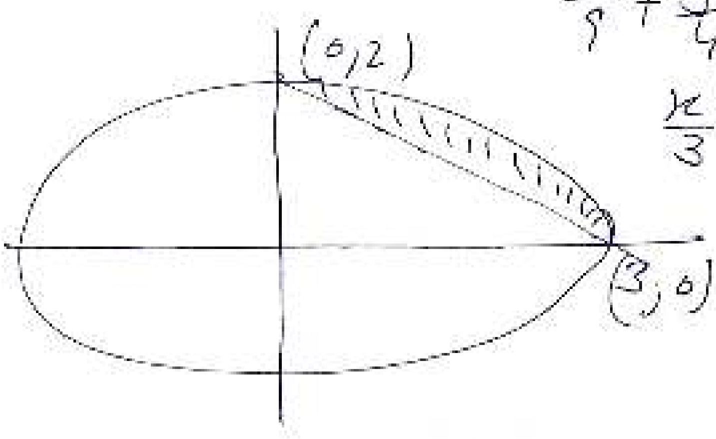
$= 2\left[\frac{x^3}{3}\right]_0^1$

$= 2\left[\frac{1}{3} - 0\right]$

$= \frac{2}{3}$ sq. unit | 1

34 or $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$\frac{x}{3} + \frac{y}{2} = 1$



$1\frac{1}{2}$

Req. Area.

$\frac{2}{3}\int_0^3 \sqrt{9-x^2}\,dx - \frac{2}{3}\int_0^3 (3-x)\,dx$ | $\frac{x^2}{9}+\frac{y^2}{4}=1$ |

$\frac{2}{3}\int_0^3 \sqrt{9-x^2}\,dx - \frac{2}{3}\int_0^3 (3-x)\,dx$	$\frac{x^2}{9}+\frac{y^2}{4}=1$ $\frac{y^2}{4}=1-\frac{x^2}{9}$ $y^2=\frac{4}{9}(9-x^2)$	$1\frac{1}{2}$
$\frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$ $-\frac{2}{3}\left[3x-\frac{x^2}{2}\right]_0^3$	$y=\frac{2}{3}\sqrt{9-x^2}$ $\frac{x}{3}+\frac{y}{2}=1$ $\frac{y}{2}=1-\frac{x}{3}$ $\frac{y}{2}=\frac{3-x}{3}$ $y=\frac{2}{3}(3-x)$	1
$\frac{2}{3}\left[+\frac{9}{2}\times\frac{\pi}{2}\right]-\frac{2}{3}\left[9-\frac{9}{2}\right]$ $\frac{\cancel{2}}{\cancel{3}}\left[+\frac{\overset{3}{\cancel{9}}\pi}{\cancel{4}2}\right]-\frac{\cancel{2}}{\cancel{3}}\times\frac{\overset{3}{\cancel{9}}}{\cancel{2}}$ $\frac{3\pi}{2}-3$ $3\left(\frac{\pi}{2}-1\right)$ sq. unit		1
		5

35

34(B) $I=\int_0^{\pi/2}(2\log\sin x-\log\sin 2x)\,dx$

$I=\int_0^{\pi/2}\log\frac{\sin^2 x}{2\sin x\cos x}\,dx$ 1

$I=\int_0^{\pi/2}\log\tan x\,dx-\int_0^{\pi/2}\log 2\,dx$ 1

$I = I_1 - \left[x \log 2\right]_0^{\pi/2}$

$I = I_1 - \frac{\pi}{2}\log 2$

$I_1 = \int_0^{\pi/2} \log \tan x \, dx$

$x \to \pi/2 - x$

$I_1 = \int_0^{\pi/2} \log \cot x \, dx$ 1

5

$2I_1 = \int_0^{\pi/2} (\log \tan x + \log \cot x)\, dx$ 1

$2I_1 = \int_0^{\pi/2} \log 1 \, dx \Rightarrow 2I_1 = 0$

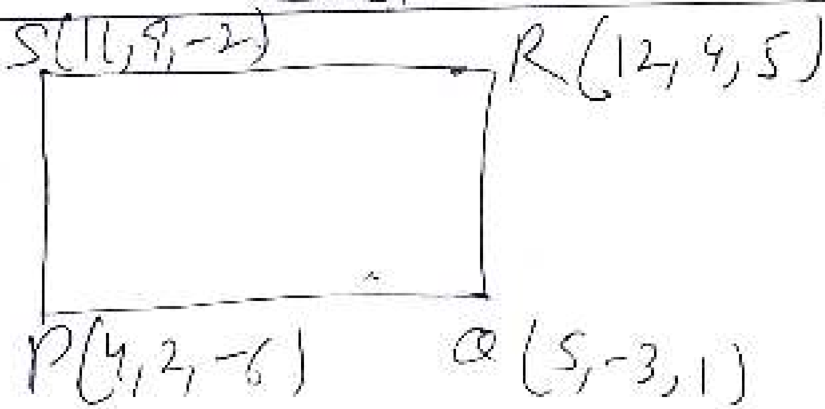
$I_1 = 0$

$\therefore I = -\frac{\pi}{2} \log 2$ 1

Diff questions of Set B

Section D

32



D. Ratio's of PR : <8, 2, 11>

D. Ratio's of QS = <6, 12, −3>

$1 < 2, 4, -1>$

Eq. of line

PR :- $\dfrac{x-4}{8} = \dfrac{y-2}{2} = \dfrac{z+6}{11} = K$ | 1

Eq. of line QS

$\dfrac{x-5}{2} = \dfrac{y+3}{4} = \dfrac{z-1}{-1} = t$ | 1

Any point of line PR

<8K+4, 2K+2, 11K−6> | 1/2

Any point of line QS

<2t+5, 4t−3, −t+1> | 1/2 | 5

for point of intersection:-

$8K+4 = 2t+5$ $2K+2 = 4t-3$

$8K-2t = 1$ $2K-4t = -5$

$\cancel{8K}-16t = -20$ | 1

$\quad + \qquad +$

$14t = 21$ $11K-6 = -t+1$

$t = \dfrac{21}{14} = \dfrac{3}{2}$ $\dfrac{11}{2}-6$ | $-\dfrac{3}{2}+1$

$t = 3/2$ $-\dfrac{1}{2}$ | $-1/2$

$2K - 4\times\dfrac{3}{2} = -5$ True

$2K = 1 \Rightarrow K = 1/2$ $\left(8, 3, -\dfrac{1}{2}\right)$ - Ans | 1

Section E

36 E: Player A hit the target

F: " B " " "

G: " C " " "

$P(E) = \frac{4}{5}$ $P(F) = \frac{3}{4}$ $P(G) = \frac{2}{3}$

(i) $P(E \cap F \cap G) = P(E) \cdot P(F) \cdot P(G)$

$= \cancel{\frac{4}{5}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{2}{\cancel{3}} = \frac{2}{5}$ 1

(ii) $P(B \cap C \cap A') = P(B) \cdot P(C) \cdot P(A')$

$= \frac{3}{4} \times \frac{2}{3} \left[1 - \frac{4}{5}\right]$ 1

$= \frac{\cancel{3}}{\cancel{4}_2} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{1}{5} = \frac{1}{10}$

(iii) $P(E \cap F \cap G') + P(F \cap G \cap E')$ 4

$+ P(E \cap G \cap F')$

$= \frac{\cancel{4}}{5} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{1}{\cancel{3}} + \frac{\cancel{3}}{\cancel{4}_2} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{1}{5}$

$+ \frac{\cancel{4}}{5} \times \frac{2}{3} \times \frac{1}{\cancel{4}}$

$= \frac{1}{5} + \frac{1}{10} + \frac{2}{15}$ 2

$= \frac{6 + 3 + 2 \times 2}{30} = \frac{13}{30}$

37

38(B) Let Prize amount for hockey $= x$

" " " " cricket $= ₹ y$

" " " " Tennis $= ₹ z$

$x + y + z = 2700$

$5x + 4y + 3z = 11000$ 1

$4x + 3y + 5z = 10700$

$$\begin{bmatrix} 1 & 1 & 1 \\ 5 & 4 & 3 \\ 4 & 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2700 \\ 11000 \\ 10700 \end{bmatrix}$$

Which is of the form.

$$AX = B$$

$|A| = 1(11) - 1(13) + 1(-7)$

$= 11 - 13 - 7 = -3 \neq 0$

$\therefore$ Equations are Consistent. 1

$$\text{adj}\,A = \begin{bmatrix} 11 & -13 & -1 \\ -2 & 1 & 1 \\ -1 & 2 & -1 \end{bmatrix}^{T}$$

$$= \begin{bmatrix} 11 & -2 & -1 \\ -13 & 1 & 2 \\ -1 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = -\frac{1}{3} \begin{bmatrix} 11 & -2 & -1 \\ -13 & 1 & 2 \\ -1 & 1 & -1 \end{bmatrix}$$

$$X = -\frac{1}{3}\begin{bmatrix} 11 & -2 & -1 \\ -13 & 1 & 2 \\ -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 2700 \\ 11000 \\ 10700 \end{bmatrix}$$

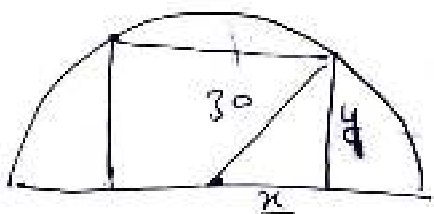
$$X = -\frac{1}{3}\begin{bmatrix} 29700 - 22000 - 10700 \\ -35100 + 11000 + 21400 \\ -2700 + 11000 - 10700 \end{bmatrix}$$

$$X = -\frac{1}{3}\begin{bmatrix} -3000 \\ -2700 \\ -2400 \end{bmatrix} = \begin{bmatrix} 1000 \\ 900 \\ 800 \end{bmatrix}$$ 2 4

$\therefore$ $x = 1000$, $y = 900$, $z = 800$

38

37(B)



$$900 = \frac{x^2}{4} + y^2$$

$$900 - \frac{x^2}{4} = y^2$$

$$\frac{3600 - x^2}{4} = y^2$$

$$y = \frac{\sqrt{3600 - x^2}}{2}$$

$$A = x \times \frac{\sqrt{3600 - x^2}}{2}$$

$$A^2 = A' = \frac{x^2}{4}(3600 - x^2)$$

$$\frac{dA'}{dx} = \frac{1}{4}\left[x^2(-2x) + (3600 - x^2)(2x)\right]$$

Put

$$\frac{dA'}{dx} = 0$$

$$-2x^3 + 7200x - 2x^3 = 0$$

$$7200x = 4x^3$$

$$x^2 = \frac{7200}{4}$$

$$x^2 = 1800$$

$$x = \sqrt{1800}$$

$$x = 30\sqrt{2}$$

$$\frac{d^2A'}{dx^2} = \frac{1}{4}\left[-6x^2 + 7200 - 6x^2\right]$$

$$= \frac{1}{4}\left[7200 - 12x^2\right]$$

At $x = 30\sqrt{2}$ $2\frac{1}{2}$

$$\frac{d^2A'}{dx^2} = \frac{1}{4}\left[7200 - 12(1800)\right] < 0$$

$\therefore$ $x = 30\sqrt{2}$ is the point of maxima

Remaining Area

$$\frac{1}{2} \times (30)^2 \pi - \frac{30\sqrt{2} \times \sqrt{3600 - 1800}}{2}$$ $1\frac{1}{2}$

$$450\pi - \frac{900 \times \cancel{2}}{\cancel{2}}$$

$450(\pi - 2)$ sq. unit

Different question of
Section E of Set B

36	Blue balls : 12 Red " : 08 Yellow " : 10 Green " : 05 35		
	(i) P(First ball is blue & second ball is green) $= \frac{\cancel{12}^{6}}{\cancel{35}_{7}} \times \frac{\cancel{08}}{\cancel{34}_{17}} = \frac{6}{119}$	1	
	(ii) P(First ball is Yellow and second ball is red) $= \frac{\cancel{10}^{2}}{\cancel{35}_{7}} \times \frac{\cancel{08}}{\cancel{34}_{17}} = \frac{8}{119}$	1	4
	(iii) P(Not Yellow second ball) $= 1 - \frac{10}{34} = \frac{24}{34}$ P(First ball is green and second is not Yellow) $= \frac{\cancel{5}}{\cancel{35}_{7}} \times \frac{\cancel{24}^{12}}{\cancel{34}_{17}} = \frac{12}{119}$	2	