O.S.D.A.V. Public School, Kaithal
Preboard Examination(2024-25)
Class : XII
Subject : Mathematics (Core)

Time: 3 hr. **M.M.: 80**

General Instructions:-

All questions are compulsory.

(a) There are 37 questions in this question paper.
(b) SECTION A consists of 20 Multiple Choice questions.
(c) SECTION B consists of 5 questions carrying 2 marks each.
(d) SECTION C consists of 6 questions carrying 3 marks each.
(e) SECTION D consists of 4 questions carrying 5 marks each
(f) SECTION E consists of 3 case study based questions carrying 4 marks each

Section A

1.A function f: x^2-4x+5 is
(A) injective but not surjective (b) surjective but not injective
(c) both injective and surjective (d) neither injective nor surjective

2. If A= $\begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$ is a skew symmetric matrix, then value of a-2(b+c) is
(a) 0 (b) 1 (c) -10 (d) 10

3. If A is square matrix of order 3 such that the value of $|adj. A|$= 8 Then value of $|A^T|$ is
(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 8 (d) $2\sqrt{2}$

4. If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then Value of λ is
(a) -4 (b) 1 (c) 3 (d) 4

5. If $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}$ =$\begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$ then value of b-2a is
(a) 1 (b) 2 (c) -1 (d) -3

6. Find the matrix A^2, where A=$[a_{ij}]$ is a 2X2 matrix whose elements are Given by a_{ij} = maximum (i,j) – minimum (i,j)
(a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

7. The derivative of $\tan^{-1}(\frac{1+x}{1-x})$ w.r.t. x is
(a) $\frac{1}{1-x^2}$ (b) $\frac{2}{1-x^2}$ (c) $\frac{1}{1+x^2}$ (d) 0

8. If xe^y =1, then value of $\frac{dy}{dx}$ at x=1 is
(a) -1 (b) 1 (c) –e (d) $\frac{-1}{e}$

9.The function f(x)= sinx+cosx , $0< x < \frac{\pi}{2}$ has a local maximum at x =
(a) 0 (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

10.Given a curve y= 7x- x^3 and x increases at the rate of 2 units/sec. The rate at which the slope of the curve is changing ,when x=5 is
(a) -60 units/sec (b) 60 units/sec (c) -70units/sec (d) -136 units/sec

11. $\int \frac{-\cos 2x}{\sin^2 x \cos^2 x}\,dx$

(a) cotx-tanx+c (b) tanx-cotx+c (c) –cotx-tanx+c (d) cotx +tanx+c

12. $\int_0^{\frac{\pi}{2}} e^x(\sin x - \cos x)dx$

(a) 0 (b) 1 (c) -1 (d) 2

13. The area of region bounded by the curve y = $\frac{1}{x}$, the x-axis and between x=1 and x=6 is

(a) $\frac{5}{6}$ (b) $\frac{1}{6}$ (c) log_e^6 (d) log_e^5

14. If p and q are the degree and order of the differential equation

$(\frac{d^2y}{dx^2})^2+3\frac{dy}{dx}+\frac{d^3y}{dx^3}$=4 then value of 2p-3q is

(a)-7 (b) 7 (c) -3 (d) 2

15.If ABCD is a parallelogram and AC and BD are its diagonals , then

$\overrightarrow{AC} + \overrightarrow{BD}$ is

(a) $2\overrightarrow{DA}$ (b) $2\overrightarrow{AB}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{BD}$

16.The angle which the line $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ makes with the positive direction

Of Z –axis is

(a) $\frac{5\pi}{6}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

17. Direction cosines of line 5x-3 = 15y+7 =3-10z are

(a) $\frac{3}{7},\frac{4}{7},\frac{2}{7}$ (b) $\frac{6}{7},\frac{2}{7},\frac{3}{7}$ (c) $\frac{-2}{7},\frac{-6}{7},\frac{3}{7}$ (d) $\frac{6}{7},\frac{2}{7},\frac{-3}{7}$

18. IF A and B are events such that P($\frac{A}{B}$)= P($\frac{B}{A}$) $\neq 0$ then

(A) A$\subset B$,but A$\neq B$ (b) A=B (c) A$\cap B = \phi$ (d) P(A)=P(B)

ASSERTION – REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a Statement of Reason (R). Choose the correct out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion (A):The principal value of $\cot^{-1}(\sqrt{3})$is $\frac{\pi}{6}$

Reason(R) : Domain of $\cot^{-1} x$ is R - [-1,1]

20.Assertion(A) : The Cartesian equation of a line passing through (4,3,2) and (1,1,1) is $\frac{x-1}{3} = \frac{y-1}{2} = \frac{z-1}{1}$

Reason(R) : Cartesian equation of a line passing through (a,b,c) and (d,e,f) is given By $\frac{x-d}{a-d} = \frac{y-e}{b-e} = \frac{z-f}{c-f}$

Section B

21. Find value of K if $\tan^{-1}[k \sin(2\cos^{-1}\frac{\sqrt{3}}{2})]=\frac{\pi}{3}$

22. Find the value of k so that given function f(x)= $\begin{cases} \frac{e^{3x}-e^{-5x}}{x} & if\ x \neq 0 \\ k & if\ x = 0 \end{cases}$

Is continuous at x=0

Or

Check for differentiability of the function f defined by f(x) = $|x-3|$ at the point x=3

23. The area of the circle is increasing at a uniform rate of 2 cm^2/sec. How fast is the circumference of the circle increasing when the radius r=5 cm?

24. $\int \frac{tan^2x \quad sec^2x}{1-tan^6x}\, dx$

Or

$\int \frac{dx}{\sqrt{3-2x-x^2}}$

25. If a line makes an angle α, β, γ with the coordinate axes, then find the value of cos2α+ cos2β + cos2γ

Section C

26. Find $\frac{dy}{dx}$, if $(cosx)^y$ = $(cosy)^x$

Or

If $\sqrt{1-x^2}$ +$\sqrt{1-y^2}$ = a(x-y), prove that $\frac{dy}{dx}$ = $\sqrt{\frac{1-y^2}{1-x^2}}$

27. If x = a$sin^3\theta$, y = b $cos^3\theta$, then find $\frac{d^2y}{dx^2}$ at θ = $\frac{\pi}{4}$

28. Evaluate $\int_0^{\pi} \frac{e^{cosx}}{e^{cosx}+e^{-cosx}}\, dx$

Or

Find $\int_{-2}^{1} |x^3-x|$ dx

29.Find the particular solution of the differential equation

$\frac{dy}{dx}$ – 2xy = 3x^2 e^{x^2} ; y(0) =5

Or

Find the general solution of the differential equation

(x^3+y^3) dy = x^2y dx

30. Find the value of p so that the lines l_1 : $\frac{1-x}{3} = \frac{7y+14}{p} = \frac{z-3}{2}$

And l_2: $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other. Also find the equation of a line passing through a point (3,2,-4) and parallel to l_1

31. If A and B are two independent events ,then prove that the probability

of occurrance of atleast one of A and B is given by 1-P(A')P (B')

Section- D

32. IF A = $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations :

x+2y -3z =1 ; 2x-3z =2 ; x +2y =3

Or

Find the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ and hence

Solve system of linear equations x+2y -3z =-4 ; 2x+3y +2z = 2 ; 3x- 3y- 4z = 11

33. Find the area bounded by circle $x^2 + y^2 = 16$ and the line $\sqrt{3}$ y = x in the first quadrant ,using integration

34.Find the length and the foot of perpendicular drawn from the point (2,-1,5)

On the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$ Also find the image of this point

Or

Find the shortest distance between the lines L_1 and L_2 given below:

L_1 : The line passing through (2, -1,1) and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$

L_2 : $\vec{r} = \hat{\imath} + (2\mu + 1)\hat{\jmath} - (\mu + 2)\hat{k}$

35. Solve the following L.P.P graphically Maximise Z= 60 x +40 y

Subject to x+2y $\leq$ 12

2x+y $\leq$ 12 ; 4x +5y $\leq$ 20 ; x,y $\geq$ 0

Section –E

36. According to recent research, air turbulence has increased in various Regions around the world due to climate change . Turbulance makes flightsBumpy and often delays the flights. Assume that, an aeroplane observes severe turbulence , moderate turbulence or light turbulence with equal probabilities. Furthur ,the chance of an airplane reaching late to the destination are 55%, 37% and 17%due to severe, moderate and light turbulence respectively On the basis of the given information ,answer the following questions

(i) Find the probability that an airplane reached its destination late
(ii) If the airplane reached its destination late, find the probability that It was due to moderate turbulence

37. A housing society wants to commission a swimming pool for its residents. For this they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 m^3. Cost of land is Rs. 500 per square meter. The cost of digging increases with depth and cost for the whole pool is Rs. 4000

Suppose the side of square plot is x metres and depth is h metres. On The basis of above information answer the following question

(i) Write Cost C(h) as a function in terms of h.

(ii) Find critical point

(iii) Use second derivative test to find the values of h for which

Cost of constructing the pool is minimum. What is the minimum cost of construction of the pool

Or

Use first derivative test to find depth of pool so that cost of construction cost is minimum. Also find relation between x and h for minimum cost

38.

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Sachin and Damini are playing Ludo at home during summer vacations. While rolling the dice ,Sachin's father Satish observed and noted the possible .Outcomes of the throw every time belongs to the set {1,2,3,4,5,6}. Let A be the set of players while B be the set of all possible outcomes.

A = {S,D} , B = {1,2,3,4,5,6}

(i) defined on set B by R={(x,y): y is divisible by x}

(ii) Show that relation R is reflexive and transitive but not symmetric

Let R be a relation on B defined by R= {(1,2),(2,2),(1,3),(3,4),(3,1),(4,3),(5,5)} .Then check whether r is an equivalence relation.

(iii) Satish wants to know the number of functions from A to B . How many no. of functions are possible?

Or

Satish wants to know the number of relations possible from A to B .How man y number of relations are possible

Pre-Board Examination (2024-25)

Mathematics (Core)

Marking Scheme / Hints to Solutions.

Note :- Any other relevant answer not given here in but given by students are suitably awarded.

Q.NO.	Value Points / Key Points	Marks allotted to each key point	Total points
1.	(d) Neither injective nor surjective	1	1
2.	(a) 0	1	1
3.	(d) $2\sqrt{2}$	1	1
4.	(d) 4	1	1
5.	(d) -3	1	1
6.	(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	1	1
7.	(c.) $\frac{1}{1+x^2}$	1	1
8.	(a.) -1	1	1
9.	(c.) $\frac{\pi}{4}$	1	1
10.	(d.) -136 units / sec	1	1
11.	(d) $\cot x + \tan x + C$	1	1
12.	(b.) 1	1	1
13.	(c.) $\log_e^6$	1	1
14.	(a) -7	1	1
15.	(c.) $2\overrightarrow{BC}$	1	1

16.	(c.) $\pi/6$	1	1
17.	(d.) $\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}$	1	1
18.	(d.) $P(A) = P(B)$	1	1
19.	(c) A is true but R is false.	1	1
20.	(a.) Both A and R are true and R is correct explanation of A.	1	1
21.	$\tan^{-1}\left[k\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}$ $\tan^{-1}\left[k\sin\left(\cancel{2}\times\frac{\pi}{\cancel{6}\,3}\right)\right]$ $\tan^{-1}\left[k\sin\frac{\pi}{3}\right]$ $\tan^{-1}\left[k\times\frac{\sqrt{3}}{2}\right] = \tan^{-1}\sqrt{3}$ $k\times\frac{\cancel{\sqrt{3}}}{2} = \cancel{\sqrt{3}}$ $k = 2$	1/2 1/2 1	2
22.	$f(x) = \lvert x-3\rvert$ $f(x) = \begin{cases} (x-3), & x>3 \\ -(x-3), & x<3 \end{cases}$ L.H.D. $\lim_{x\to3^-}\frac{f(x)-f(3)}{x-3}$ $\lim_{x\to3^-}\frac{-(x-3)-0}{x-3}$ $\lim_{x\to3^-}\frac{-\cancel{(x-3)}}{\cancel{(x-3)}}$ $=-1$ R.H.D. $\lim_{x\to3^+}\frac{f(x)-f(3)}{x-3}$ $\lim_{x\to3^+}\frac{x-3-0}{x-3}$ $\lim_{x\to3^+}\frac{\cancel{(x-3)}}{\cancel{(x-3)}}$ $=1$ L.H.D. $\neq$ R.H.D. $\therefore f(x)$ is not differentiable at $x=3$ OR	1 (L.H.D) 1 (R.H.D)	2

(22) $f(x) = \begin{cases} \frac{e^{3x} - e^{-5x}}{x} & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$

$\lim_{x \to 0} \frac{e^{3x} - e^{-5x}}{x}$

$\lim_{x \to 0} \frac{e^{-5x}(e^{8x} - 1) \times 8}{8x}$ 1/2

$\lim_{x \to 0} 8e^{-5x}$ 1/2 2

$= 8 \times e^{0}$

$= 8 \times 1 = 8$

$f(0) = k$

$f(x)$ is continuous at $x = 0$, $\lim_{x \to 0} = f(0)$ 1

$\therefore k = 8$

23. $A \to$ Area of circle

$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$ $\frac{dA}{dt} = 2\,cm^2/sec$

$C \to$ circumference of circle 1/2

$C = 2\pi r$

$\frac{dC}{dr} = 2\pi$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ $\frac{dC}{dt} = \frac{dC}{dr} \times \frac{dr}{dt}$ 2

$\cancel{2} = \cancel{2}\pi r \times \frac{dr}{dt}$ $= 2\cancel{\pi} \times \frac{1}{\cancel{\pi} r}$ 1 1/2

$\frac{dr}{dt} = \frac{1}{\pi r}$ $= \frac{2}{r}\,cm/sec$

When $r = 5\,cm$

$\frac{dC}{dt} = \frac{2}{5} = 0.4\,cm/sec$

24. $\int \frac{\tan^2 x \sec^2 x}{1-\tan^6 x}\,dx$

$\tan^3 x = t$ 1/2

$3\tan^2 x \sec^2 x\,dx = dt$

$\frac{1}{3}\int \frac{dt}{1-t^2}$ 1/2 2

$= \frac{1}{3}\left[\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|\right] + C$ 1

$= \frac{1}{6}\log\left|\frac{1+\tan^3 x}{1-\tan^3 x}\right| + C$

Or

(b) $\int \frac{dx}{\sqrt{3-2x-x^2}}$

$-(x^2+2x-3+1-1)$

$-\left[(x+1)^2-(2)^2\right]$ 1

$= (2)^2-(x+1)^2$

$= \int \frac{dx}{\sqrt{(2)^2-(x+1)^2}}$ 2

$= \sin^{-1}\frac{x+1}{2} + C$ 1

25. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\frac{\cos 2\alpha + 1}{2} + \frac{\cos 2\beta + 1}{2} + \frac{\cos 2\gamma + 1}{2} = 1$ 1

$\cos 2\alpha + 1 + \cos 2\beta + 1 + \cos 2\gamma + 1 = 2$ 2

$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 2-3$

$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$ 1

26. $(\cos x)^y = (\cos y)^x$

Taking log both sides

$\log(\cos x)^y = \log(\cos y)^x$ 1

$y\log(\cos x) = x\log(\cos y)$

Differentiating both sides

$y\cdot\frac{(-\sin x)}{\cos x} + \log(\cos x)\frac{dy}{dx} = x\cdot\frac{(-\sin y)}{\cos y}\frac{dy}{dx} + \log(\cos y)$ 1

$\log(\cos x)\frac{dy}{dx} + x\tan y\frac{dy}{dx} = \log(\cos y) + y\tan x.$

$\frac{dy}{dx}\left(\log(\cos x) + x\tan y\right) = \log(\cos y) + y\tan x$

$$\frac{dy}{dx} = \frac{\log(\cos y) + y\tan x}{\log(\cos x) + x\tan y}$$

OR.

(b) $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

$x = \sin\theta \quad y = \sin\phi$

$\sqrt{\cos^2\theta} + \sqrt{\cos^2\phi} = a(\sin\theta - \sin\phi)$

$\cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$

$\cancel{2}\cos\cancel{\frac{\theta+\phi}{2}}\cos\frac{\theta-\phi}{2} = \cancel{2}a\cos\cancel{\frac{\theta+\phi}{2}}\sin\frac{\theta-\phi}{2}$

$\cot\frac{\theta-\phi}{2} = a$

$\theta - \phi = 2\cot^{-1}a$ 3

~~Differentiation~~

$\sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$ 1

Differentiating both sides

$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$

$\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

Hence, proved.

27. $x = a\sin^3\theta \qquad y = b\cos^3\theta$

$\frac{dx}{d\theta} = a.3\sin^2\theta\cos\theta \qquad \frac{dy}{d\theta} = -b.3\cos^2\theta.\sin\theta$

$\frac{dx}{d\theta} = 3a\sin^2\theta\cos\theta \qquad \frac{dy}{d\theta} = -3b\cos^2\theta\sin\theta$ 1

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{dy}{dx} = \frac{-\cancel{3}b\cos^{2}\theta\,\cancel{\sin\theta}}{\cancel{3}a\sin^{2}\theta\,\cancel{\cos\theta}} = -\frac{b}{a}\cot\theta$$

$$\frac{d^2y}{dx^2} = \frac{b}{a}(\operatorname{cosec}^2\theta)\frac{d\theta}{dx} \qquad 1$$

$$= \frac{b}{a}\operatorname{cosec}^2\theta \times \frac{1}{3a\sin^2\theta\cos\theta}$$

$$= \frac{b}{3a^2}\operatorname{cosec}^4\theta\sec\theta \qquad 3$$

$$\frac{d^2y}{dx^2} \text{ at } \theta = \frac{\pi}{4} \Rightarrow \frac{b}{3a^2}(\sqrt{2})^4 \times \sqrt{2} \qquad 1$$

$$= \frac{4\sqrt{2}\,b}{3a^2}$$

28. $$I = \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}\,dx \quad \text{——— (1)}$$

$$x = \pi - x$$

$$I = \int_0^{\pi} \frac{e^{\cos(\pi-x)}}{e^{\cos(\pi-x)} + e^{-(\cos(\pi-x))}}\,dx \qquad 1$$

$$I = \int_0^{\pi} \frac{e^{-\cos x}}{e^{-\cos x} + e^{\cos x}}\,dx \quad \text{——— (2)}$$

Adding (1) & (2)

$$2I = \int_0^{\pi} \frac{\cancel{(e^{\cos x} + e^{-\cos x})}}{\cancel{(e^{\cos x} + e^{-\cos x})}}\,dx \qquad 1 \qquad 3$$

$$2I = \int_0^{\pi} 1\,dx$$

$$2I = [x]_0^{\pi}$$

$$2I = \pi \qquad 1$$

$$I = \frac{\pi}{2}.$$

$\int_{-2}^{1} |x^3 - x| \, dx \Rightarrow \int_{-2}^{1} |x(x-1)(x+1)| \, dx$

$= \int_{-2}^{-1} (+x)(1-x)(x+1) \, dx + \int_{-1}^{0} (-x)(1-x)(1+x) \, dx + \int_{0}^{1} x(x+1)(1-x) \, dx$ 1

$= \int_{-2}^{-1} x(1-x^2) \, dx + \int_{-1}^{0} -x(1-x^2) \, dx + \int_{0}^{1} x(1-x^2) \, dx$

$= \int_{-2}^{-1} x - x^3 \, dx + \int_{-1}^{0} x^3 - x \, dx + \int_{0}^{1} x - x^3 \, dx$ 1 3

$= \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{-2}^{-1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{1}$

$= \left[\left(\frac{1}{2} - \frac{1}{4}\right) - (2-4)\right] + \left[0 - \left(\frac{1}{4} - \frac{1}{2}\right)\right] + \left[\frac{1}{2} - \frac{1}{4}\right]$ 1

$= \frac{1}{4} + 2 + \frac{1}{4} + \frac{1}{4} = \frac{3}{4} + 2 = \frac{11}{4}$

29. $\frac{dy}{dx} - 2xy = 3x^2 e^{x^2}$

$I.F = e^{\int -2x \, dx}$

$= e^{-\cancel{2}x^2/\cancel{2}}$

$= e^{-x^2}$ 1

Solⁿ :-

$y \cdot I.F = \int Q \cdot I.F \, dx + C$

$y \cdot e^{-x^2} = \int 3x^2 \cancel{e^{x^2}} \cdot \cancel{e^{-x^2}} \, dx + C$

$y \cdot e^{-x^2} = \frac{\cancel{3}x^3}{\cancel{3}} + C$ $1\frac{1}{2}$

$y \cdot e^{-x^2} = x^3 + C$ 3

at $x = 0$, $y = 5$

$5(1) = (0)^3 + C$

$C = 5$ $\frac{1}{2}$

Solⁿ :- $y \cdot e^{-x^2} = x^3 + 5$

OR $(x^3+y^3)\,dy = x^2y\,dx$

$$\frac{dy}{dx} = \frac{x^2y}{x^3+y^3}$$

$$y = vx$$

$$\frac{dy}{dx} = v + \frac{dv}{dx}x$$ 1

$$v + x\frac{dv}{dx} = \frac{x^2(vx)}{\cancel{}x^3+v^3x^3}$$

$$v + x\frac{dv}{dx} = \frac{v\cancel{x^3}}{\cancel{x^3}+v^3\cancel{x^3}}$$

$$x\frac{dv}{dx} = \frac{v}{1+v^3} - v$$

$$x\frac{dv}{dx} = \frac{\cancel{v}-\cancel{v}-v^4}{1+v^3}$$

$$x\frac{dv}{dx} = -\frac{v^4}{1+v^3}$$ 1

$$\int\frac{1+v^3}{v^4}dv = -\int\frac{1}{x}dx$$

$$\int v^{-4}+\frac{1}{v}\,dv = -\log|x| + c$$ 3

$$-\frac{v^{-3}}{3} + \log v = -\log|x| + c$$

$$\log v + \log x = \frac{v^{-3}}{3} + c$$

$$\log\frac{y}{x} + \log x = \frac{x^3}{3y^3} + c$$ 1

$$\log y = \frac{x^3}{3y^3} + c$$

30. $l_1 : \frac{1-x}{3} = \frac{7y+14}{p} = \frac{z-3}{2}$

$l_1 : \frac{x-1}{-3} = \frac{y+2}{1/7p} = \frac{z-3}{2}$ 1/2

$l_2 : \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$

$l_2: \frac{x-1}{-3/7\,p} = \frac{y-5}{1} = \frac{z-6}{-5}$

$l_1 \perp l_2$

$-3 \times -\frac{3}{7}p + \frac{1}{7}p \times 1 + 2\times(-5) = 0$

$\frac{9}{7}p + \frac{1}{7}p - 10 = 0$ | 1

$\frac{10}{7}p = 10$

$p = \frac{70}{10} = 7$

Passing point $\rightarrow (3, 2, -4)$ | 1/2

Req. line is $\parallel$ to l_1 | 3

So, dir^n cosines of l_1 = dir^n cosines of req. line

$= \langle -3, 1, 2 \rangle$

Eq^n of req line $\rightarrow$ | 1

$\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2}$

31. Given :- $P(A\cap B) = P(A).P(B)$

To prove :- $P(A\cup B) = 1 - P(A')P(B')$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= P(A) + P(B) - P(A)P(B)$

$= P(A)(1-P(B)) + P(B)$

$= P(A)P(B') + P(B)$ | 3

$= P(A)P(B') + 1 - P(B')$

$= P(B')(P(A)-1) + 1$

$= 1 - P(A')P(B')$

32.(b) $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$ $\quad B = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ 15 & 9 & -1 \end{bmatrix}$

$AB = \begin{bmatrix} -6+28+45 & 17+10-27 & 13-16+3 \\ -12+42-30 & 34+15+18 & 26-24-2 \\ -18-42+60 & 51-15-36 & 39+24+4 \end{bmatrix}$ | 1

$$AB = \begin{bmatrix} 67 & 0 & 0 \\ 0 & 67 & 0 \\ 0 & 0 & 67 \end{bmatrix} = 67I$$

$x + 2y - 3z = -4$

$2x + 3y + 2z = 2$ 1

$3x - 3y - 4z = 11$

$$\begin{array}{ccccc} A & & X & & C \\ \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} & & \begin{bmatrix} x \\ y \\ z \end{bmatrix} & = & \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} \end{array}$$

$X = A^{-1}C$ 1

$AB = 67I$

Pre multiplying by A^{-1}

$A^{-1}AB = 67A^{-1}I$

$B = 67A^{-1}$

$A^{-1} = \frac{1}{67}B$ 1

$$X = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$X = \frac{1}{67}\begin{bmatrix} 24 + 34 + 143 \\ -56 + 10 - 88 \\ 60 + 18 - 11 \end{bmatrix}$$ 5

$$X = \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$ 1

$x = 3, \; y = -2, \; z = 1$

(a) $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$

$\text{adj } A = \begin{bmatrix} 6 & -3 & 4 \\ -6 & 3 & 0 \\ -6 & -3 & -4 \end{bmatrix}' = \begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}$ 1

$|A| = 1(6) - 2(3) - 3(4)$

$= 6 - 6 - 12 = -12$

$A^{-1} = -\frac{1}{12} \begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}$ 1

To find :

$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ 1

$AX = B$

Pre multiplying by A^{-1}

$A^{-1}AX = A^{-1}B$ 1

$X = A^{-1}B$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{12} \begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ 5

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{12} \begin{bmatrix} 6-12-18 \\ 3+6-9 \\ 4-0-12 \end{bmatrix}$ 1

$x = 2, \; y = \frac{1}{2}, \; z = \frac{2}{3}$

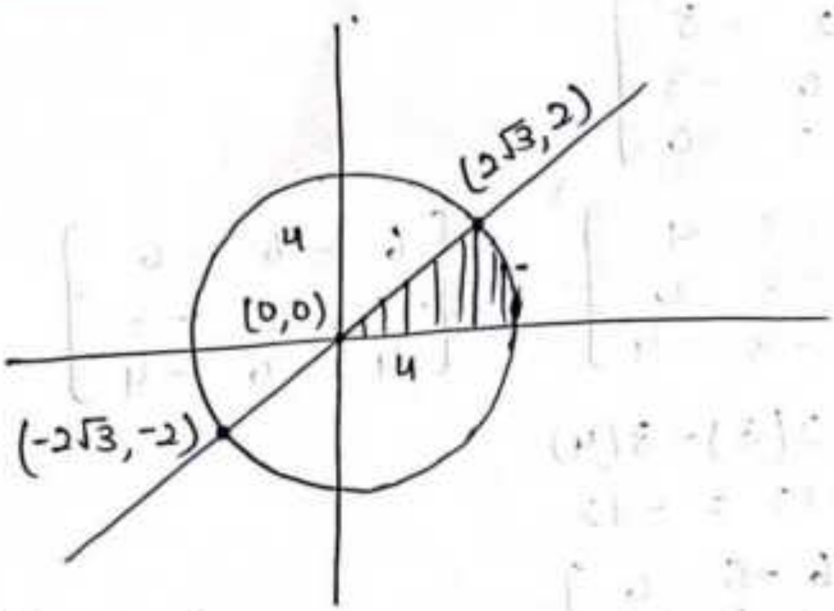
33. $x^2 + y^2 = 16$ $\qquad \sqrt{3}y = x$

$3y^2 + y^2 = 16$ $\qquad \Longrightarrow (2\sqrt{3}, 2)$

$4y^2 = 16$ $\qquad (-2\sqrt{3}, -2)$ 1

$y^2 = 4$

$y = \pm 2$



1

Req. Area →

$\frac{1}{\sqrt3}\int_0^{2\sqrt3} x\,dx + \int_{2\sqrt3}^{4} \sqrt{16-x^2}\,dx$ 1

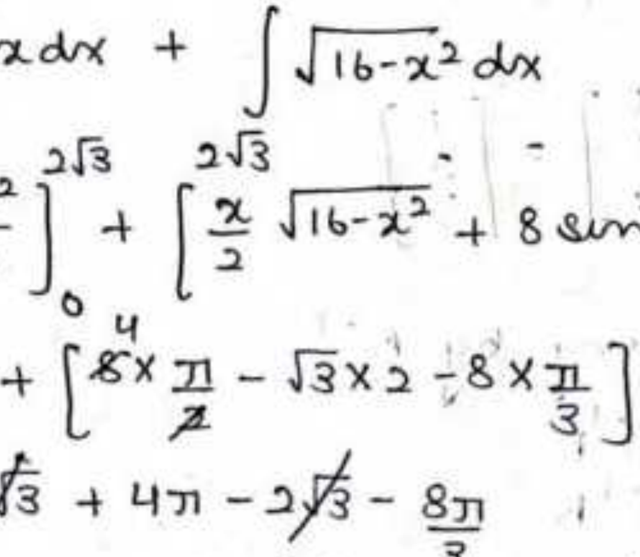
$\frac{1}{\sqrt3}\left[\frac{x^2}{2}\right]_0^{2\sqrt3} + \left[\frac{x}{2}\sqrt{16-x^2} + 8\sin^{-1}\frac{x}{4}\right]_{2\sqrt3}^{4}$ 1 5

$\frac{1}{\sqrt3}(6) + \left[8\times\frac{\pi}{\cancel{2}} - \sqrt3\times2 - 8\times\frac{\pi}{3}\right]$

$2\cancel{\sqrt3} + 4\pi - 2\cancel{\sqrt3} - \frac{8\pi}{3}$ 1

$\Longrightarrow \frac{4\pi}{3}$ sq. units

34



$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-1}$ — ①

Any point on line ① → $(10k+11, -4k-2, -11k-8)$ 1

Dirⁿ cosines of AB → $\langle 10k+11-2, -4k-2+1, -11k-8-5 \rangle$

$\Rightarrow \langle 10k+9, -4k-1, -11k-13 \rangle$

Eq^n of AB →

$$\frac{x-2}{10k+9} = \frac{y+1}{-4k-1} = \frac{z-5}{-11k-13}$$

AB ⊥ line 1

$10(10k+9) - 4(-4k-1) - 11(-11k-13) = 0$

$100k + 90 + 16k + 4 + 121k + 143 = 0$ 1

$237k + 237 = 0$

$k = -1$

$B \Rightarrow (1,2,3)$

$AB = \sqrt{(2-1)^2 + (-1-2)^2 + (5-3)^2}$ 1

$= \sqrt{(1)^2 + (-3)^2 + (2)^2} = \sqrt{1+9+4} = \sqrt{14}$

$AB = \sqrt{14}$ units

C is the image of A. 1

B is the mid point of AC. 5

$\frac{\alpha+2}{2} = 1$ $\frac{\beta-1}{2} = 2$ $\frac{\gamma+5}{2} = 3$

$\alpha = 2-2$ $\beta = 4+1$ $\gamma = 6-5$ 1

$\alpha = 0$ $\beta = 5$ $\gamma = 1$

Co-ordinates of C → $(0,5,1)$

(b) line passing through $(2,-1,1)$ & parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$

$l_1 : (2\hat{i} - \hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} + 3\hat{k})$ 1

$l_2 : (\hat{i} + \hat{j} + 2\hat{k}) + \mu(2\hat{j} - \hat{k})$

$(\vec{a_2} - \vec{a_1}) = -\hat{i} + 2\hat{j} + \hat{k}$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 3 \\ 0 & 2 & -1 \end{vmatrix}$$ 2

$= \hat{i}[-1-6] - \hat{j}[-1] + \hat{k}[2]$

$= -7\hat{i} + \hat{j} + 2\hat{k}$

$$\vec{d} = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$ 2 5

$$= \left| \frac{7+2+2}{\sqrt{49+1+4}} \right|$$

$$= \frac{11}{\sqrt{54}} \text{ units}$$

35. $Z = 60x + 40y$

$x + 2y \leq 12$

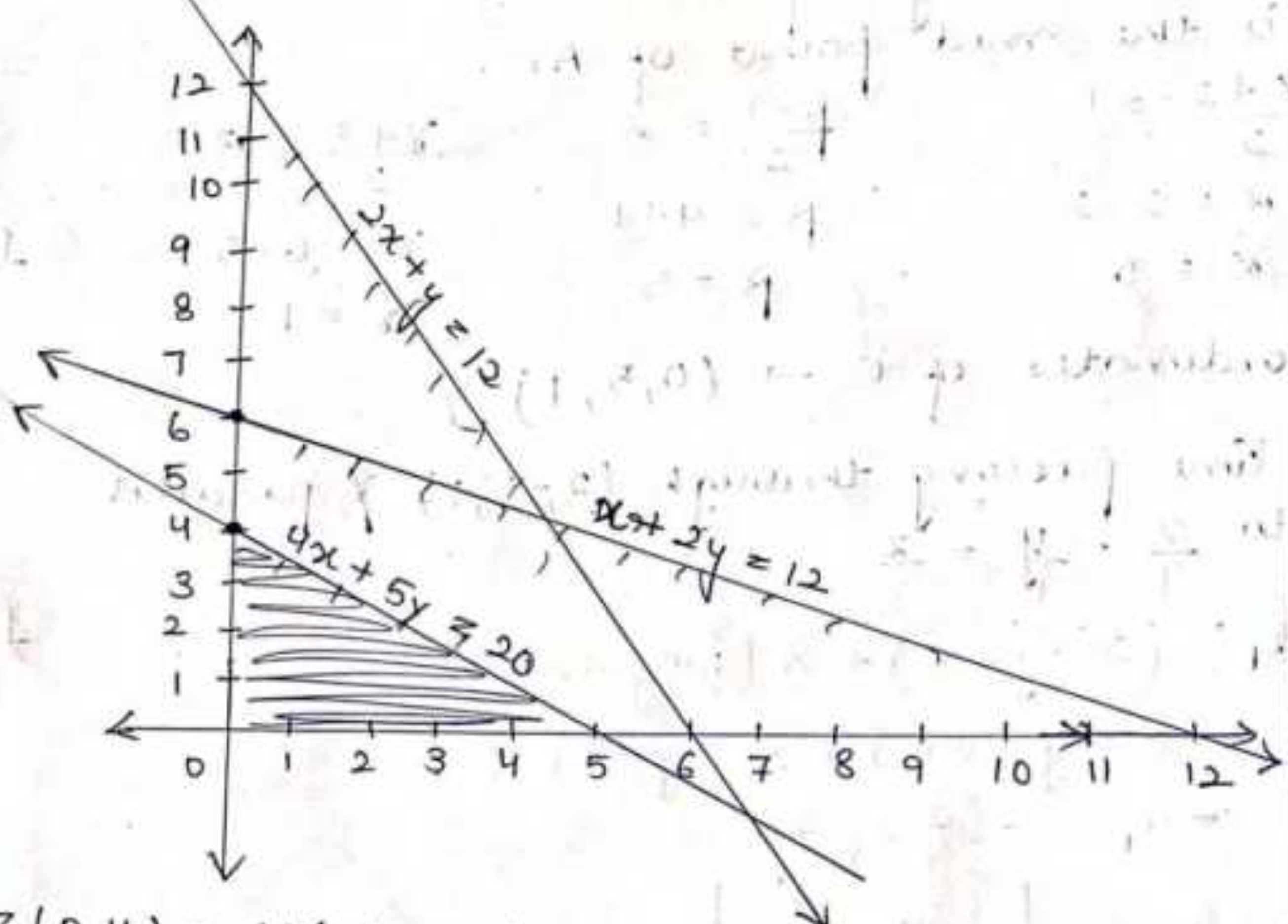
x	0	12
y	6	0

$2x + y \leq 12$ $\frac{1}{2}$

x	0	6
y	12	0

$4x + 5y \leq 20$

x	0	5
y	4	0



2

5

$Z(0,4) = 60(0) + 40(4) = 160$

$Z(5,0) = 60(5) + 40(0) = 300$ 1

Z is maximum at $x = 5$, $y = 0$

& max value is 300.

36. E_1: There is severe turbulence.
E_2: There is moderate turbulence.
E_3: There is light turbulence.

$P(E_1) = \frac{1}{3} \quad P(E_2) = \frac{1}{3} \quad P(E_3) = \frac{1}{3}$

A: Airplane reached late

$P(A|E_1) = 55/100$

$P(A|E_2) = 37/100$

$P(A|E_3) = 17/100$

(i) $P = P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)$

$= \frac{1}{3} \times \frac{55}{100} + \frac{1}{3} \times \frac{37}{100} + \frac{1}{3} \times \frac{17}{100}$ 2

$= \frac{109}{300}$ 4

(ii) $P(E_2|A) = \dfrac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)}$ 2

$= \dfrac{\frac{1}{3} \times \frac{37}{\cancel{3}00}}{\frac{109}{300}} = \frac{37}{109}$

37. length of swimming pool $= x$ m
breadth of swimming pool $= x$ m
let depth of swimming pool $= h$ m

A.T.Q.

$x^2 h = 250$

$x^2 = \frac{250}{h}$ 1

Cost of land $= 500x^2$

$= 500 \times \frac{250}{h}$

$= \frac{125000}{h}$

Cost of digging $= 4000h^2$

(i) $C(h) = \frac{125000}{h} + 4000h^2$

(ii) $C'(h) = -\frac{125000}{h^2} + 8000h$

For C.V.

$C'(h) = 0$

$-\frac{125000}{h^2} + 8000h = 0$

$\cancel{8000}h = \frac{125\cancel{000}}{h^2}$

$h^3 = \frac{125}{8}$ | 1

$h = \frac{5}{2} = 2.5$

(iii) $C''(h) = 125000 \times \frac{2}{h^3} + 8000$ | 4

$C''(h)$ is +ve for $h = 2.5$

$\therefore h = 2.5$ is the point of local minima

Minimum Cost $= \frac{125000}{5} \times 2 + 4000\left(\frac{25}{4}\right)$

$= 50000 + 25000$ | 2

$= ₹75000$

38. (i) $R = \{(x,y) : y \text{ is divisible by } x\}$

$(a,a) \in R \ \forall \ a \in B$

a is divisible to itself

$\therefore$ R is reflexive

$(a,b) \in R \ \forall \ a,b \in R$

b is divisible by a

a is factor of b | 2

$\therefore (b,a) \notin R$

$\therefore$ R is not symmetric.

if $(a,b) \in R$ & $(b,c) \in R \ \forall \ a,b,c \in B$

b is divisible by a

c is divisible by b

c is divisible by a

$\therefore (a, c) \in R$

Hence, R is transitive

(ii) $R = \{(1,2), (2,2), (1,3), (3,4), (3,1), (4,3), (5,5)\}$

$(a,a) \notin R \ \forall \ a \in B$

$\therefore$ R is not reflexive | 1

Hence, R is not an equivalence relation | 4

(iii) No. of functions $\Rightarrow 36$

OR | 1

No. of relations $\Rightarrow 2^{12}$